Vol. 15, No. 01, pp. 388-402, 2025 DOI: 10.23960/jpp.v15i1.pp388-402

Jurnal Pendidikan Progresif

e-ISSN: 2550-1313 | p-ISSN: 2087-9849
http://jurnal.fkip.unila.ac.id/index.php/jpp/

Didactic Transposition of Absolute Value in Calculus Courses: Analyzing Curricula, Textbooks, and Classroom Instruction

M. Azhari Panjaitan, Turmudi, & Jarnawi Afgani Dahlan*
Department of Mathematics Education, Universitas Pendidikan Indonesia, Indonesia

*Corresponding email: jarnawi@upi.edu

Received: 12 February 2025 *Accepted: 24 February 2025* *Published: 20 April 2025*

Abstract: Didactic Transposition of Absolute Value in Calculus Courses: Analyzing Curicula, Textbooks, and Classroom Instruction. Objective; This study aimed to analyze the knowledge to be taught in absolute value learning as part of the didactic transposition process**. Method;** A qualitative approach with a descriptive design was employed to examine curriculum documents and calculus textbooks as primary data sources. The research adopted a didactic transposition and praxeological framework to analyze the structure of absolute value instruction. The methodology included document selection based on inclusion criteria, content analysis of absolute value concepts in textbooks, comparison with theoretical praxeology models, and expert validation to ensure alignment with established mathematical and pedagogical principles. Triangulation was applied by analyzing multiple textbooks and curricula to enhance validity and reliability. **Findings;** The study results indicate that learning absolute value requires comprehensive material to bridge the transition of knowledge from school to university. Differential calculus textbooks introduce the absolute value concept with an initial definition as a non-negative value that geometrically represents distance, followed by a piecewise definition commonly found in high school textbooks. Content analysis reveals that task techniques related to absolute value material predominantly focus on perceptual, algebraic, and operational aspects, while graphical visualization is lacking, which hinders a thorough conceptual understanding. This highlights the contrast between the procedural approach in schools and the conceptual approach in universities, emphasizing the necessity of comprehensive materials to prevent learning obstacles. **Conclusion;** The findings emphasize the need for a more comprehensive presentation of absolute value in university calculus textbooks to support a smooth transition from school to higher education. The analysis revealed that while absolute value is introduced conceptually, the lack of graphical representation limits students' understanding. Additionally, inconsistencies between curriculum documents and learning materials indicate the necessity of aligning instructional resources. To prevent learning obstacles, it is recommended that absolute value content in differential calculus textbooks be presented in a complete and structured manner, integrating both procedural and conceptual approaches.

Keywords: knowledge to be taught, absolute value, didactic transposition.

To cite this article:

Panjaitan, M. A., Turmudi, & Dahlan, J. A. (2025). Didactic Transposition of Absolute Value in Calculus Courses: Analyzing Curricula, Textbooks, and Classroom Instruction. *Jurnal Pendidikan Progresif*, *15*(1), 388-402. doi: 10.23960/jpp.v15i1.pp388-402.

INTRODUCTION

Absolute value is one of the fundamental concepts in mathematics, playing a crucial role from secondary school to advanced university-level mathematics (Aziz et al., 2019). This concept is not merely a computational tool but also a fundamental idea in understanding distance (Ellis & Bryson, 2011), magnitude, and the properties of real numbers (Ponce, 2008; Taylor & Mittag, 2015). Furthermore, the absolute value is deeply connected to the development of more complex mathematical topics, such as functions, limits, and inequalities (Almog & Ilany, 2012; Isnaini, 2022; Novianti & Shodikin, 2018), all of which are extensively explored in university-level differential calculus courses.

A firm conceptual grasp of absolute value significantly aids students in various branches of mathematics. However, numerous studies indicate that students struggle to understand and apply the concept appropriately. Students often face difficulties when solving problems related to absolute value, particularly in applying the concept of distance and its definition (Amir, 2017; Jupri et al., 2022). Research by Cahyaningtyas et al. (2021) further suggests that students' errors in solving absolute value equations and inequalities stem from a lack of conceptual understanding. Amir (2017) also, many students rely on memorizing formulas rather than understanding underlying mathematical principles, leading to frequent problem-solving errors.

Conceptual challenges in learning absolute value are not limited to students but also reflect broader issues in mathematics education. Many students struggle with the concept, resulting in errors in solving absolute value problems, equations, and inequalities, as well as in algebraic manipulations involving absolute value (Adinda et al., 2021; Almog & Ilany, 2012; Elia et al., 2016; Panjaitan, 2023; Serhan & Almeqdadý, 2018). These difficulties are exacerbated by misconceptions among mathematics teachers regarding the formal construction and application of absolute value. Amani and Mawarsari (2019) found that students' errors are not solely due to their misunderstandings but are also influenced by external factors, particularly the effectiveness of teachers in delivering mathematical concepts. Insufficient explanations and misconceptions from teachers hinder students' ability to develop a deep conceptual understanding, causing them to rely heavily on rote memorization.

Beyond teacher-related factors, the quality of absolute value learning is also shaped by external influences such as the curriculum and textbooks used as references (Cahyani et al., 2024). A dense or unclear curriculum, coupled with textbooks that lack detailed conceptual explanations, can pose significant obstacles to students attempting to understand abstract mathematical concepts such as absolute value. While previous studies have examined students' difficulties in understanding absolute value, research analyzing how this concept is presented in university-level calculus curricula and textbooks remains scarce. Most existing studies have focused on students' procedural errors (Amir, 2017; Cahyaningtyas et al., 2021). without exploring how the didactic transposition process influences how absolute value is taught. Therefore, this study aims to fill this research gap by analyzing how absolute value is structured in the curriculum and textbooks and how these elements affect students' conceptual understanding at the university level.

To gain a comprehensive understanding of the factors influencing students' comprehension of absolute value, this study employs the didactic transposition framework (Bosch & Gascón, 2006). This framework is used to analyze how a mathematical concept of absolute value, in this case, is transformed from its pure mathematical form into a form that is accessible for classroom instruction (Chevallard & Bosch, 2020). Didactic transposition in mathematics education was first introduced by (Chevallard, 1992; Chevallard & Johsua, 1985), who classified mathematics into two forms: an object or body of knowledge and a subject to be taught and learned. In this study, didactic transposition is applied to examine how the concept of absolute value is transformed from its scientific form into its classroom presentation. The stages of didactic transposition used in this research include external transposition, which analyzes how absolute value is structured in the curriculum and textbooks as teachable knowledge; internal transposition, which investigates how lecturers deliver absolute value instruction and how students construct their understanding; and praxeological analysis, which identifies the types of tasks, problem-solving techniques, and justifications for these techniques within textbooks and classroom instruction. This process is illustrated in Figure 1, which depicts the transformation from scholarly knowledge to learned knowledge through different stages of didactic transposition.

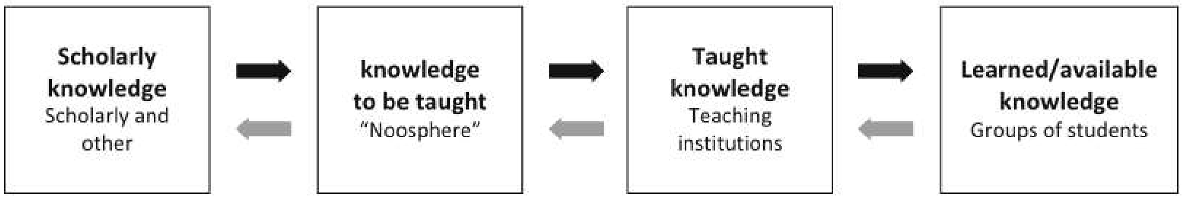

Figure 1. The process of didactic transposition (Chevallard & Bosch, 2014)

In addition to the didactic transposition framework, this study employs the praxeological approach, which is part of the Anthropological Theory of Didactics (ATD) (Suryadi, 2019). Praxeology ($\wp$) consists of four components: types of tasks (T), which analyze the nature of exercises in textbooks; techniques(τ), which describe methods for solving problems; technology (θ) which explains the rationale behind these techniques; and theory(Θ), which justifies the technology. This structure is schematically represented as $\wp = [T / \tau / \theta / \Theta]$ (Chevallard & Bosch, 2020; Topphol, 2023).

Topphol (2023) highlighted the issue of visual representations (logos) used in textbooks to unify different mathematical concepts, noting that these representations often confuse students rather than aid understanding. His research found that abstract concepts presented with ambiguous visual models lead to significant comprehension difficulties. This aligns with Balacheff (2023), who emphasized a paradigm shift in mathematics education, where proof is increasingly recognized as essential to understanding mathematical concepts. On the other hand, reference books used for learning absolute value often contain errors and insufficient explanations, further complicating students' learning experiences (Panjaitan & Juandi, 2024).

Based on previous research and the theoretical framework outlined above, this study aims to explore the didactic transposition of absolute value and how it shapes the knowledge to be taught in calculus courses. By analyzing the structure of absolute value in curricula and textbooks and how it is presented in university classrooms, this study provides insights into the strengths and limitations of existing instructional practices and suggests improvements for better conceptual understanding.

■ METHOD

This study employed a qualitative approach with a descriptive design to investigate how absolute values are presented in higher education curricula and calculus textbooks. The research aimed to analyze the alignment between textbook content and mathematical principles and identify potential learning difficulties related to absolute value. The methodology adopted in this study consists of four main sections: Participants, Research Design and Procedures, Instruments, and Data Analysis.

Participants

The participants in this study were documents, including higher education curricula and calculus textbooks, which were analyzed to examine the presentation of absolute value. The selection of documents was based on their relevance to the subject matter and their use in higher education institutions. The inclusion criteria consisted of calculus textbooks used in university-level mathematics courses and curriculum documents outlining the intended learning outcomes for absolute value. The exclusion criteria included textbooks that do not cover absolute value and curricula that do not specify mathematical content related to absolute value.

Research Design and Procedures

The research process was carried out in several stages: first, relevant curriculum documents and textbooks were identified and collected as primary data sources. Second, a praxeological model for teaching absolute value was developed based on theoretical mathematical structures, forming a reference framework for analysis. Third, content analysis was conducted by comparing the reference praxeology with the presentation of absolute value in textbooks and curricula, focusing on categorizing and identifying key themes related to absolute value in mathematical and pedagogical contexts. Fourth, validation was performed by consulting experts in mathematics education to ensure the alignment of the findings with established mathematical principles and appropriate pedagogical practices. Finally, the interpretation phase involved analyzing inconsistencies between the theoretical model and the actual presentation of the concept, identifying potential learning difficulties, and evaluating the alignment with the didactic transposition process (Chevallard & Bosch, 2014; Chevallard & Johsua, 1985). The research was conducted for four months to ensure thorough analysis and accurate verification of results.

Instruments

The primary instrument used in this study was document analysis, structured within a framework adapted from Chevallard and Johsua (1985) didactic transposition and praxeology models. The analysis was conducted based on the following indicators: the types of tasks presented in textbooks, techniques for solving absolute value problems, and the technological and theoretical explanations provided. Triangulation was applied to ensure validity and reliability by comparing multiple textbooks and curricula, while an expert review process was carried out to validate the accuracy of interpretations (Wijayanti & Winslow, 2017).

Data Analysis

The data analysis was conducted through the didactic transposition approach, applying a praxeology model that includes the type of task, completion technique, technology, and theoretical framework used (Chevallard & Bosch, 2014). A reference praxeology model for absolute value was created and compared with the praxeology elements in the calculus textbooks (Wijayanti & Winslow, 2017). This comparison allowed researchers to evaluate whether the presentation of the material in the textbooks was consistent with mathematical principles and whether the instructional design could support students' understanding of absolute value.

■ RESULT AND DISCUSSION

The place or position of the concept of absolute value in the mathematics learning process is analyzed through the curriculum of the mathematics education program and differential calculus textbooks. Thus, the textbook analysis uses the theory of praxeology to discover how the textbook presents and develops the concept of absolute value.

Absolute Value Material in The Curriculum of The Study Program

In the study program curriculum section, two documents were reviewed: the curriculum prepared by the study program curriculum team and the course distribution of the Mathematics

Education study program at one of the universities in Medan. However, the two documents could not be found online through the website or offline by asking the Mathematics Education study program team directly. Based on this, the two documents were only reviewed through the RPS (Semester Study Plan) of the mathematics education study program courses and the KRS (Study Plan Card) of the semester 1 students in the mathematics education study program.

The first document is based on the RPS of the differential calculus course. In the document, there is an explanation of CPMK (course learning outcomes) and sub-CPMK (course learning outcomes) from 1 to 5, but there is no explanation of differential calculus as a compulsory or optional course. Based on the CPMK and sub-CPMK in Figures 2 and 3, the concept of absolute value is in the first position in the real number system material.

Table 1. CPMK (course learning outcomes)

Capaian Pembelajaran Mata Kuliah (CPMK)	***Deskripsi***
CPMK1	*Mahasiswa mengenal dan menguasai konsep sistem bilangan real, nilai mutlak dan pertidaksamaan dalam memecahkan masalah sains*
CPMK2	*Mahasiswa mengenal dan menguasai konsep limit fungsi dan kekontinuan dalam memecahkan masalah sains*
CPMK3	*Mahasiswa mengenal dan menguasai konsep turunan fungsi dalam memecahkan masalah sains*
CPMK4	*Mahasiswa mengenal dan menguasai penggunaan turunan untuk menyelesaikan masalah yang berkaitan dengan turunan fungsi*
CPMK5	*Mahasiswa mengenal dan menguasai konsep anti turunan dan integral tak tentu agar dapat digunakan untuk memahami masalah kalkulus lebih lanjut*

Table 2. Sub CPMK (course learning outcomes)

Kemampuan Akhir Tiap Tahapan Belajar (Sub-CPMK)	
Sub-CPMK	***Deskripsi***
Sub-CPMK1	*Mahasiswa mampu menggunakan konsep sistem bilangan real, nilai mutlak dan pertidaksamaan dalam memecahkan masalah sains*
Sub-CPMK2	*Mahasiswa mampu menggunakan konsep limit fungsi dan kekontinuan dalam memecahkan masalah sains*
Sub-CPMK3	*Mahasiswa mampu menggunakan konsep turunan fungsi dalam memecahkan masalah sains*
Sub-CPMK4	*Mahasiswa mampu menggunakan penggunaan turunan untuk menyelesaikan masalah yang berkaitan dengan turunan fungsi*
Sub-CPMK5	*Mahasiswa mampu menggunakan konsep anti turunan dan integral tak tentu agar dapat digunakan untuk memahami masalah kalkulus lebih lanjut*

This research focuses on the concept of absolute value in the differential calculus course. This absolute value material is taught after a basic understanding of the real number system. In the course description (see Figure 4), the materials studied are the real number system and functions, limit and continuity of functions, derivatives of functions and differentials, and the use of derivatives that underlie applied mathematics in various fields. On the other hand, these materials are clarified in the study material section in Figure 4, namely, there is material for Chapter 1 (real number system, inequality and absolute value, real functions), chapter 2 (function limit, function continuity, infinite limit, indefinite form of function limit), chapter 3 (derivatives and rules for determining them, higher order derivatives, implicit derivatives), chapter 4 (use of derivatives: maximum and minimum, monotonicity and skewness, local maximum and minimum, graphing functions with calculus, real problems about maximums and minima), and Chapter 5 (antiderivative, indefinite integral). Based on the order of the material in the study material, absolute value is studied according to the real number system and inequality. These two materials become reference materials as prerequisite materials that must be mastered before studying absolute value. This is the foundation for understanding absolute value. However, the course description does not detail the prerequisite topics that must be mastered before learning absolute value material. With a simple assumption, the existence of introductory material before learning absolute value will assist in mastering the concept of absolute value.

An approach that combines procedural and conceptual knowledge can help reinforce students' comprehension and deepen their understanding of absolute value. Fatimah et al. (2020) contextual, conceptual, and procedural knowledge play key roles in solving mathematical tasks. Students who can link contextual knowledge with conceptual knowledge tend to perform better in solving mathematical tasks. This approach introduces procedures and connects the understanding of concepts to broader contexts.

Table 3. Short description of the course

Deskripsi Singkat MK
Mata kuliah Kalkulus I merupakan mata kuliah yang mempelajari tentang sistem bilangan real dan fungsi, limit dan kekontinuan fungsi, turunan fungsi dan diferensial, serta penggunaan turunan yang mendasari terapan matematika di berbagai bidang dan menjadi dasar bagi topik-topik matematika lebih lanjut.

Table 4. Study materials and sources for books

Bahan Kajian: Materi Pembelajaran	*1. Sistem bilangan real, pertidaksamaan dan nilai mutlak, fungsi real* *2. Limit fungsi, kekontinuan fungsi, limit tak hingga, bentuk tak tentu limit fungsi* *3. Turunan dan aturan menentukan turunan, turunan tingkat tinggi, turunan implisit* *4. Penggunaan Turunan: maksimum dan minimum, kemonotonan dan kecekungan, maksimum dan minimum lokal, grafik fungsi dengan kalkulus, masalah nyata tentang maksimum dan minimum* *5. Anti turunan, integral tak tentu*
Pustaka	

Utama:	*1.*	*Varberg, Purcell, Rigdon (2007). Calculus, Ninth Edition, Pearson Education Internasional, USA*
Pendukung:	*1.*	*James Stewart (2010). Calculus, Seventh Edition. Brooks/Cole. Belmont, USA*
	2.	*Koko Marlono (1999). Kalkulus. Erlangga, Jakarta, Indonesia*

The teaching materials used during learning consist of main and supporting sources (see Figure 5). However, during learning practice in the classroom, the teaching materials used differed from those written in the RPS, namely the teaching materials written by the team of calculus lecturers. The teaching materials used during the learning process are analyzed praxeologically in the Absolute Value Material section of the textbook.

The second document is the distribution of courses in the Mathematics Education program from semester 1 to semester 8. However, the college did not find this second document offline or online. Referring to these problems, the researchers looked at the contents of the study plan cards of students who were studying calculus courses, so they were found in semester 1 students, as shown in Figure 6 below.

No	Kode MK	Nama Matakuliah	SKS	KLS	Hari	Pukul	Ruang	Blok
1	1MDK40002	PERKEMBANGAN PESERTA DIDIK	2	B	Senin	08:00-09:39	77.1.08	1
					Rabu	08:00-09:39	192.2.02	
2	2MPA40001	MATEMATIKA DASAR	3	B	Senin	09:41-11:20	192.4.01	1
					Selasa	09:41-11:20	77.1.08	
3	3MAT40001	FISIKA DASAR	3	B	Rabu	11:21-12:59	12.1.09	1
					Kamis	14:41-16:19	192.2.01	
4	3MAT40003	HIMPUNAN DAN LOGIKA	3	B	Senin	13:01-15:30	192.2.02	1
					Selasa	13:01-15:30	77.1.01	
5	1MDK40001	FILSAFAT PENDIDIKAN	2	B	Senin	08:00-09:39	12.1.09	2
					Rabu	08:00-09:39	12.1.09	
6	1UNM40001	KEPEMIMPINAN	2	B	Kamis	09:41-11:20	192.4.01	2
					Jum'at	08:00-09:40	192.4.02	
7	2MPA40002	TEKNOLOGI INFORMASI DAN LITERASI DATA	2	B	Selasa	15:31-17:10	192.2.02	2
					Rabu	15:31-17:10	192.2.02	
8	3MAT40002	KALKULUS DIFERENSIAL	3	B	Selasa	13:01-15:30	77.1.01	2
					Jum'at	13:51-16:20	192.5.10	
SKS Maksimal: 24			Jumlah SKS / Matakuliah : 20/8					

Figure 2. 1st semester student study plan card

The study plan map view has a block column filled with numbers 1 and 2. Block 1 indicates that the course will be studied at the first to eighth session, and Block 2 indicates that the course will be studied at the ninth to tenth session. Each class will differ in the use of blocks 1 and 2, but the courses in the first semester are still the same and are listed on each student's study plan card, as shown in Figure 6. It is only the determination of blocks 1 and 2 that will be different from class to class. The differential calculus course listed on the study plan card weighs 3 credits in block 2. It can be seen that block 1 on the study plan card of introductory mathematics courses can be a prerequisite for differential calculus courses on absolute value material. On the other hand, in the first chapter of the differential calculus course, the real number system and inequality become supporting prerequisites for absolute value. Thus, the courses in Block 1 and Block 2 are interrelated in supporting the absolute value prerequisite in differential calculus learning.

The course of differential calculus is a new thing for first-year mathematics students because learning in college is a transition period from school institutions to university institutions with different learning systems. Mathematics teaching in school institutions on absolute value material is often only about procedural understanding due to the limited time to complete materials in mathematics learning. On the other hand, curriculum changes in schools also affect the differences, such as the shift from the 2013 curriculum to the independent curriculum, which has the effect that absolute value material is not compulsory in grade X. However, in simple terms, absolute value material has been introduced to junior high school students in grade 7 in number material. Thus, the material learned in lectures differs from the material learned in school because the material is studied more deeply, conceptually and procedurally, with prior knowledge and experience. Through this learning process, students can remember and relearn concepts learned in school and develop new knowledge.

The explanation of the review of the two documents, namely the curriculum and course distribution of the mathematics education program, has provided support and benefits in understanding the concept of absolute value in differential calculus courses. However, the explanation described is still general and brief. The learning of absolute value becomes more focused and directed through the guide. This is consistent with supporting the understanding of the concept of absolute value broadly and deeply through references related to absolute value and the support given by instructors to students.

Absolute Value Material in The Textbook

The semester study plan prepared by the Differential Calculus teaching team has included several source books used as references for learning differential calculus, namely Calculus, Ninth Edition by Varberg et al. (2013), Calculus, Seventh Edition by Stewart (2016), and Calculus by Martono (1999). However, the textbook used while learning differential calculus differs from what is written in the semester study plan. The textbook used is compiled by the differential calculus teaching team, so this book is the subject of the research to be analyzed using praxeology. The primary materials contained in the differential calculus book are the real number system, limit and continuity, derivatives, and the use of derivatives. The absolute value material is found in the first chapter, namely the real number system, after the subchapter on inequalities.

In the first chapter, there is the real number system material with subchapters and sub-materials, namely the real number system, inequality, absolute value, one-variable functions, and trigonometric functions and their inverses. In the introductory chapter, the absolute value material is studied after the real number system and inequality material. This is the prerequisite material for the absolute value material. In the differential calculus book, there are two sub-materials, namely, the definition of absolute value and inequalities involving absolute value. On the other hand, the explanation of absolute value in this book still does not facilitate an in-depth understanding of the concept of absolute value. Based on the examination of this book, there is no graphical presentation or in-depth understanding of the concept and complete working procedures. Therefore, the textbook of differential calculus is analyzed in terms of textbook content using the praxeological approach. The content analysis of the book with praxeology includes the presentation of material and tasks based on concepts, as well as examples of problems and exercises listed in this book.

Definition of Absolute Value

At the beginning of the presentation of absolute value material in the differential calculus book, it is emphasized that absolute value is the basis for understanding the concept of the limit of a function. The first understanding presented is that absolute value is a real number x that is always non-negative. On the other hand, the notion of absolute value is viewed geometrically; namely, the absolute value of a real number 5ØeÜ describes the distance between the number 5ØeÜ and the zero point on the number line, without regard to direction. This value shows how far the number 5ØeÜ is from zero and can be visualized with an illustration as the distance between two points on the number line. Based on the illustration presented in the book, we can define absolute value.

The preliminary explanation of this material is the definition of absolute value, along with a number line drawing that shows the definition. The absolute value material in college needs to be studied more deeply to understand the concept and solution procedure. In the differential calculus book, some theorems can be proved so that the understanding of the concept of absolute value and the solution of absolute value problems is correct. Therefore, this book presents only partial proofs of the theorems contained in the book.

This book's first explanation aims to gradually understand the concept of absolute value. However, the sentences presented are still incomplete in explaining the compatibility in everyday life with absolute value in mathematics. This shows that there are differences in the presentation of material in school institutions and higher education institutions. School institutions pay more attention to contextual and procedural understanding, while higher education institutions pay attention to the whole, namely conceptual, contextual and procedural understanding. Based on this, the textbooks used in differential calculus learning are analyzed with a praxeological approach, as shown in Table 1 below.

Table 1. A praxeological analysis of the concept of the absolute value

Task (T)	**Technique (τ)**	**Technology (θ)**	**Theory (Θ)**
T_1 Find the solution to the following! 1. $\|2\| =$ 2. $\|-5\| =$ 3. $\|3-\sqrt{12}\| =$	τ_1 *Perceptual* and *operational* Determine the absolute value of each number.	θ_1 Through the understanding of absolute value geometrically can determine the absolute value of each number	Θ_1 Application of the notion of absolute value
T_2 By using the definition of absolute value 4. $\|x-1\| =$	τ_2 *Perceptual and algebraic* Determine absolute value solutions through the use of definitions.	θ_2 Used the definition of absolute value to determine the problem's solution.	Θ_2 Understanding the definition of absolute value.
T_3 Prove theorem 1.3.2 below! 1. For every real number x and y, hold $\|x\| = \|y\|$ if and only if $x = \pm$ and $x^2 = y^2$ 2. If $a \geq 0$, so $\|x\| \geq a$ if and only if $x \geq a$ $or\ x \leq -a\ \ and\ x^2 \geq a^2$ 3. For every real number x and y, hold: • $\|x-y\| \leq \|x\| + \|y\|$ • $\|x\| - \|y\| \leq \|x-y\|$ • $\|\|x\| - \|y\|\| \leq \|x-y\|$	τ_3 *perceptual and algebraic* Prove theorem 1.3.2 by using understanding of the concept of absolute value.	θ_3 Through understanding the concept of absolute value to prove theorem 1.3.2	Θ_3 Theorem proving (absolute value properties)

4. For every real number x and y, hold: • $\lvert xy\rvert \le \lvert x\rvert . \lvert y\rvert$ • $\left\lvert \frac{x}{y} \right\rvert \le \frac{\lvert x\rvert}{\lvert y\rvert}, y \neq 0$			
T_4 Using the absolute value property, determine the following solutions. • $\lvert x-1\rvert > 2$ • $\lvert x-1\rvert < 2$	τ_4 *perceptual, operational, and algebraic* Determine absolute value solutions using theorem 1.3.2	θ_4 Using theorem 1.3.2 to determine the absolute value solution.	Θ_4 Application of the properties of absolute value

In the content analysis of the book using the praxeological approach, the notion of absolute value has four tasks (*T*) that have been observed. Each task is determined by the technique (*t*), technology (*q*) and theory (*Q*) presented in the book. The first task leads to the knowledge of the notion of absolute value geometrically. On the other hand, it leads to the definition of absolute value, which will help to solve the second problem. Through these two problems, the definition of absolute value is presented step by step during the learning process from the college institution to the school institution, as follows . $\lvert x\rvert = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$. Based on the process of the two tasks, the way students think in understanding the concept of absolute value can be guided. However, the presentation in the book is still incomplete, namely, the graphical visualization of the concept of absolute value, as shown in Figure 1. Graphic visualization is very helpful in constructing knowledge about understanding the concept of absolute value from colleges. Thus, understanding the concept of absolute value is not only procedural.

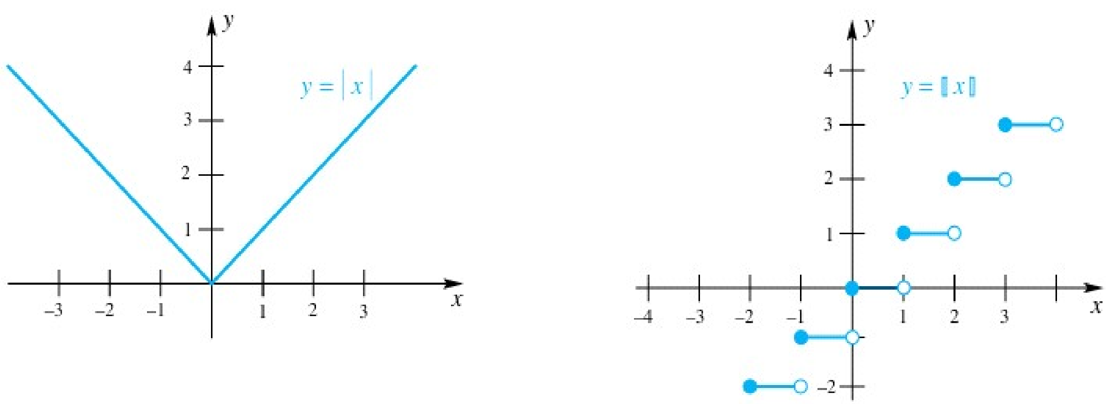

Figure 3. Graph y = $\lvert x\rvert$ addapted by Stewart (2016)

The third and fourth tasks are the proofs of theorems, or often called the properties of absolute value. The properties of absolute value are very helpful in solving absolute value problems. There are many ways to solve absolute value other than using the definition of absolute value. This strongly encourages students to use different techniques to solve for absolute value. The fourth problem is an example of using the properties of absolute value in the solution. By using theorem 3, which is for every $x, y \in R$, so $-|x| \leq x \leq |x|$ and $-|y| \leq y \leq |y|$ consequently obtained $-(|x| + |y|) \leq x + y \leq |x| + |y|$. Then the correct answer is $|x-1| < 2 \Leftrightarrow -2 < x - 1 < 2 \Leftrightarrow -1 < x < 3$ and $|x-1| > 2 \Leftrightarrow x - 1 > 2 \text{ or } x - 1 < -2 \Leftrightarrow x > 3 \text{ or } x < -1$. The properties of absolute value can be explained to senior high school students to help them understand the concept of absolute value.

Inequalities with Absolute Values

Introducing inequalities with absolute values begins by converting the algebraic form into a form that does not contain absolute values. For inequalities, on the other hand, we first have to remember the inequality. In Martono's book (1999), there is an introduction to inequalities, namely the general form of an algebraic inequality with one real variable is $\frac{A(x)}{B(x)} < \frac{C(x)}{D(x)}$, A, B, C, D polynomial. The < sign can be replaced by >, d", or e" in these forms. The set of all real numbers x that satisfy the inequality is called the set of inequality solutions. The standard procedure for solving inequalities is to use elementary algebraic formulas and sequences, converting the form to $\frac{P(x)}{Q(x)} < 0$, with terms P and Q, decomposing P and Q into positive definite linear and/or quadratic factors, determining the sign of the inequality on the number line, and determining the solution set and visualizing it in the form of a number line interval. Next, analyze the book's content using praxeology again concerning inequalities involving absolute values, as in Table 2 below.

The first task in the inequalities containing absolute values sub-material has directed students to be able to convert algebraic forms into forms that do not contain absolute values. The second task is applying absolute value inequalities by determining the solution for each given problem. The technique (*t*) for both tasks uses perceptual, operational, and algebraic methods. The solution to the problem uses the properties of absolute value. The following is the solution to the first problem by changing the algebraic form to one

Table 2. Praxeological analysis of absolute value inequalities

Task (T)	Technique (τ)	Technology (θ)	Theory (Θ)
T_1 Change the algebraic form $3\|x\| + \|x-2\|$ into a form that does not contain absolute values	τ_1 *perceptual, operational, and algebraic* Transform algebraic forms into forms that do not contain absolute values	θ_1 Through solving each case or using one of the theorems 1.3.2, you can transform the algebraic form into a form that does not contain absolute values.	Θ_1 Inequalities involving absolute values
T_2 Find the Settlement Set (HP) of the following inequality! • $\|x-5\| \leq 4$ • $\|x+1\| < 2x-7$ • $\|8-3x\| < \|2x\|$ • $x^2\|x\| \leq 8$ • $\|x-2\| + 2\|x-1\| > 1$	τ_2 *perceptual, operational, and algebraic* Determine the solution set of inequalities using the concept of absolute value and its properties.	θ_2 Through understanding the concept of absolute value and its properties to get the solution set of the inequality.	Θ_2 The set of solutions to the inequality

that does not contain absolute values, namely

$$3|x| + |x-2| = \begin{cases} -4x+2, & x < 0 \\ 2x+2, & 0 \le x < 2 \\ 4x-2, & x \ge 2 \end{cases}$$

After that, the solution to second problem is as in 2a . $|x-5| \le 4 \Leftrightarrow -4 \le x-5 \le 4 \Leftrightarrow 1 \le x \le 9$. So, the solution set of the following inequality is [1,9]. For the other two problems, you can work with the definition of absolute value or the properties of absolute value.

Comparison of Procedural and Conceptual Approaches in Teaching Absolute Value

This study found that the conceptual approach is more effective in enhancing students' understanding of the absolute value concept than the procedural approach. Students who were taught using the conceptual approach demonstrated better abilities to apply the concept of absolute value in various contexts, including real-world problem-solving. This finding aligns with the research of Stupel and Ben-Chaim (2014), which shows that the graphical representation of absolute value equations helps students understand solutions more deeply and grasp the concept better .

The conceptual approach enables students to understand the relationships between mathematical concepts, improving long-term retention and enhancing problem-solving skills. In contrast, the procedural approach often focuses on the steps to follow without fostering a deep understanding of the underlying meaning of those procedures, limiting students' ability to apply their knowledge to new situations. This is supported by Fatimah et al. (2020), who emphasize that students' misconceptions typically arise from a shallow understanding of the absolute value concept. This issue can be addressed with a more conceptual teaching approach .

Moreover, research by Jupri et al. (2022) demonstrates that using interactive software, such as GeoGebra, in teaching absolute value equations significantly enhances conceptual understanding and procedural fluency. This aligns with local studies, such as Malinda and Hidayati (2022), which found that students' conceptual ability to solve absolute value equations is greatly influenced by their understanding of the mathematical theories and models that underlie them .

By comparing these findings with international research, it becomes clear that a deeper conceptual understanding, through a more conceptual teaching approach, provides students with better tools to tackle mathematical challenges in broader contexts. This indicates that the conceptual approach not only strengthens students' grasp of the specific topic of absolute value but also equips them with the skills to handle more complex mathematical problems in the future.

CONCLUSION

The absolute value of the knowledge to be taught has been analyzed through several sources, namely the curriculum of the study program and the differential calculus textbook, the source of calculus learning materials. Some of the findings obtained are the absence of the curriculum of the Mathematics Education study program online or offline and the book sources listed on the study learning plan with the book during learning. The curriculum is analyzed through the study learning plan and student study plan card, while the calculus textbook prepares the differential calculus lecturer team. Through absolute value learning, it is necessary to present and teach material comprehensively so that the transition of knowledge from school institutions to higher education institutions is by the concept of absolute value to be taught. The course description contained in the semester study plan is by the materials contained in the differential calculus textbook. This is consistent with the learning process of understanding the concept of absolute value. However, the learning of absolute value still looks different between colleges and schools, which are only concerned with procedural matters.

In the textbook presentation of differential calculus, the first introduction to the concept of absolute value is to recognize the notion of absolute value, whose value is always non-negative and geometrically known as the concept of distance. This introduction is very good for starting the emergence of the definition of absolute value. Furthermore, the definition of absolute value is introduced to the students in a piece-wise form so that this form is also presented in high school textbooks. The content analysis of the book on absolute value material with the praxeological approach found that each task given uses perceptual, algebraic, and operational techniques. However, the tasks were not found to use physical techniques. This is related to the absence of a graph as a form of visualization of the concept of absolute value, which is a shortcoming of the definition of absolute value. The next task is to prove theorems or properties of absolute value that will help solve absolute value problems. In the inequalities sub-material, there is no introduction to absolute value inequalities, so it refers to one of the books written by Koko Martono. It is strongly recommended that the differential calculus textbook writing team write the absolute value material completely so as not to raise any learning barriers that may arise. On the other hand, the problems contained in the absolute value inequality can be solved using the properties of absolute value that have been understood.

■ ACKNOWLEDGMENT

The author sincerely acknowledges the financial support provided by LPDP (Indonesia Endowment Fund for Education) under the Ministry of Finance of the Republic of Indonesia, which facilitated the completion of the doctoral program and the publication of this article. Deep gratitude is also extended to the advisors for their invaluable guidance and constructive feedback, as well as to all individuals and parties who contributed to the development and writing of this article.

■ REFERENCES

Adinda, A., PARTA, N., & Chandra, T. D. (2021). Investigation of students' metacognitive awareness failures about solving absolute value problems in mathematics education. *Eurasian Journal of Educational Research*, *95*, 17–35.

Almog, N., & Ilany, B.-S. (2012). Absolute value inequalities: High school students' solutions and misconceptions. *Educational Studies in Mathematics*, *81*, 347–364.

Amani, R. A. D. S. S., & Mawarsari, V. D. (2019). *Analisis kesulitan belajar siswa X IPS SMA Negeri 15 Semarang pada materi pertidaksamaan nilai mutlak. EDUSAINTEK*, *3*.

Amir, M. F. (2017). *Identifikasi kesulitan mahasiswa dalam memecahkan masalah open ended materi nilai mutlak. Jurnal Mercumatika/ : Jurnal Penelitian Matematika Dan Pendidikan Matematika*, *2*(2), 1–15. https://doi.org/10.26486/jm.v2i2.291

Aziz, T. A., Supiat, & Soenarto, Y. (2019). Pre-service secondary mathematics teachers' understanding of absolute value. *Cakrawala Pendidikan*, *38*(1), 203–214. https://doi.org/10.21831/cp.v38i1.21945

Balacheff, N. (2023). Notes for a study of the didactic transposition of mathematical proof. *ArXiv Preprint ArXiv:2305.18863*.

Bosch, M., & Gascón, J. (2006). Twenty-five years of the didactic transposition. *ICMI Bulletin*, *58*(58), 51–65.

Cahyani, R. G., Hadiprasetyo, K., & Wulandari, A. A. (2024). *Analisis faktor penyebab kesulitan belajar matematika siswa kelas xi sma negeri 1 nguter pada* *kurikulum merdeka. Jurnal Ilmiah Multidisiplin*, *1*(6), 511–520. https://doi.org/10.62017/merdeka

Cahyaningtyas, O., Rahardi, R., & Irawati, S. (2021). *Analisis kesalahan siswa dalam menyelesaikan soal persamaan dan pertidaksamaan nilai mutlak berdasarkan teori newman. Edumatica: Jurnal Pendidikan Matematika*, *11*(03), 104–117.

Chevallard, Y. (1992). A theoretical approach to curricula. *Journal Fuer Mathematik-Didaktik*, *13*(2), 215–230.

Chevallard, Y., & Bosch, M. (2014). Didactic transposition in mathematics education bt - encyclopedia of mathematics education (S. Lerman (ed.); pp. 170–174). Springer Netherlands. https://doi.org/10.1007/978-94-007-4978-8_48

Chevallard, Y., & Bosch, M. (2020). Didactic transposition in mathematics education. *Encyclopedia of Mathematics Education*, 214–218.

Chevallard, Y., & Johsua, M.-A. (1985). *La transposition didactique: du savoir savant au savoir enseigné*. La Pensée Sauvage,.

Elia, I., Özel, S., Gagatsis, A., Panaoura, A., & Özel, Z. E. Y. (2016). Students' mathematical work on absolute value: focusing on conceptions, errors and obstacles. *ZDM*, *48*, 895–907.

Ellis, M. W., & Bryson, J. L. (2011). A conceptual approach to absolute value equations and inequalities. *The Mathematics Teacher*, *104*(8), 592–598.

Fatimah, A. T., Zakiah, N. E., Sunaryo, Y., Gumilar, I., & Rusmana, I. (2020). Pengetahuan kontekstual, konseptual, dan prosedural siswa SMK pada pemecahan tugas konteks jangkauan. *Jurnal THEOREMS (The Original Research of Mathematics)*, *4*(2), 147–154.

Isnaini, M. (2022). *Analisis kesulitan mahasiswa dalam menyelesaikan soal pertidaksamaan, fungsi dan limit pada mata kuliah kalkulus. Jurnal Amal Pendidikan*, *3*(3), 234–241.

Jupri, A., Usdiyana, D., & Gozali, S. M. (2022). Pre-service teachers' strategies in solving absolute value equations and inequalities. *Education Sciences*, *12*(11), 743.

Malinda, T. T., & Hidayati, W. S. (2022). *Analisis kemampuan konseptual peserta didik dalam menyelesaikan soal nilai mutlak berdasarkan teori apos. Pi: Mathematics Education Journal*, *5*(1), 17–26.

Martono, K. (1999). Kalkulus. *Jakarta: Erlangga*.

Novianti, A., & Shodikin, A. (2018). *Pengembangan bahan ajar kalkulus diferensial berbasis animasi dengan pendekatan kontekstual dan kearifan lokal*. *De Fermat: Jurnal Pendidikan Matematika*, *1*(2), 12–18.

Panjaitan, M. A. (2023). *Desain didaktis pada pembelajaran persamaan dan pertidaksamaan nilai mutlak satu variabel.* Universitas Pendidikan Indonesia.

Panjaitan, M. A., & Juandi, D. (2024). Analysis of problems in learning mathematics based on difficulties, errors, and misconceptions in the material of equations and inequality absolute values of one variable: systematic literature review. *KnE Social Sciences*, 316–324.

Ponce, G. A. (2008). Using, seeing, feeling, and doing absolute value for deeper understanding. *MAtheMAtics TeAchinG in the Middle School*, *14*(4), 234–240.

Serhan, D., & Almeqdadý, F. (2018). Pre-service teachers' absolute value equations and inequalities solving strategies and errors. *The Eurasia Proceedings of*

Educational and Social Sciences, *10*, 163–167.

Stewart, J. (2016). *Calculus: early transcendentals 8th edition*. Cengage Learning.

Stupel, M., & Ben-Chaim, D. (2014). Absolute value equations–what can we learn from their graphical representation? *International Journal of Mathematical Education in Science and Technology*, *45*(6), 923–928.

Suryadi, D. (2019). *Penelitian desain didaktis (DDR) dan implementasinya*. Gapura Press Bandung.

Taylor, S. E., & Mittag, K. C. (2015). Easy absolute values? Absolutely. *Mathematics Teaching in the Middle School*, *21*(1), 49–52.

Topphol, V. (2023). The didactic transposition of the fundamental theorem of calculus. In *REDIMAT - Journal of Research in Mathematics Education* (Vol. 12, Issue 2, pp. 144–172).

Varberg, D., Purcell, E., & Rigdon, S. (2013). *Calculus: pearson new international edition*. Pearson Higher Ed.

Wijayanti, D., & Winslow, C. (2017). Mathematical practice in textbooks analysis: Praxeological reference models, the case of proportion. *REDIMAT*, *6*(3), 307–330.