

Exploring Students' Spatial Ability in Geometry Transformations Based on Cognitive Styles through the APOS Framework

Fauzan Syahbudin*, Nurjanah, & Kartika Yulianti

Department of Mathematics Education, Universitas Pendidikan Indonesia, Indonesia

*Corresponding email: fauzansyah.project@gmail.com

Received: 04 March 2026

Accepted: 09 June 2026

Published: 03 July 2026

Abstract: This study aims to analyze the spatial ability of Field Independent (FI) and Field Dependent (FD) students in two-dimensional geometric transformation tasks across three aspects: spatial visualization, mental rotation, and spatial orientation. It also examines students' cognitive development through APOS stages and identifies thinking patterns across these aspects. This qualitative case study involved six ninth-grade students from a public junior high school in Bandung, Indonesia. Participants were purposively selected based on their Group Embedded Figures Test (GEFT) scores, comprising three FI and three FD students. Data were collected through a spatial ability test and semi-structured interviews. Written responses and interview transcripts were analyzed using APOS-based coding criteria, supported by triangulation between written and verbal data. Students' spatial ability varied across cognitive styles, spatial aspects, and APOS stages. FI students generally showed higher and more varied achievement than FD students, particularly in spatial visualization. However, this pattern was not entirely consistent, as one FI student did not demonstrate sufficient evidence to be classified at the Action stage, which is the most basic stage in the APOS framework, across all aspects. Mental rotation and spatial orientation were more difficult than spatial visualization; only one FI student met the criteria for each indicator. FD students achieved mainly in spatial visualization, with two students reaching the Schema stage when visual and coordinate-based support was available. However, all FD students failed to achieve the indicators for mental rotation and spatial orientation, and mostly remained at the Action stage. This study indicates that spatial ability in geometric transformations is influenced not only by cognitive style but also by task demands, visual support, and prerequisite understanding. Geometry instruction should provide structured visual support and gradual transitions toward independent mental manipulation, particularly in rotation and orientation tasks.

Keywords: spatial ability, cognitive style, field independent, field dependent, geometry transformation, APOS theory.

Article's DOI: <https://doi.org/10.23960/jpp.v16i2.pp1279-1298>

■ INTRODUCTION

Spatial ability is an essential cognitive skill that plays a crucial role in mathematics learning, particularly in geometry, as it enables students to imagine, manipulate, and understand relationships among objects in space, both visually and mentally (Bruce et al., 2017; Gilligan-Lee & Hawes, 2022; Newcombe, 2016; Newcombe & Frick, 2010). This ability is also required in everyday activities such as reading maps, giving directions, and manipulating three-dimensional objects, making it a fundamental skill across various disciplines,

including mathematics, engineering, physics, and science (Yıldızhan & Ertekin, 2024). Furthermore, spatial ability has been identified as a strong predictor of academic achievement and career success in STEM fields, independent of mathematical and verbal abilities (Wai et al., 2009), highlighting its strategic role in learning and cognitive development.

Spatial ability consists of several interrelated yet distinct aspects, each with different cognitive demands. Recent spatial reasoning frameworks emphasize that spatial ability should not be treated

as a single, general construct, as spatial visualization, mental rotation, and spatial orientation may support different forms of mathematical activity (Adams et al., 2023; Harris, 2023; Ramful et al., 2017). This view is consistent with earlier classifications of spatial ability. Maier (1998) categorized spatial ability into multiple components, while Maresch (2013) simplified these into three main aspects: spatial visualization, mental rotation, and spatial orientation, which are considered relevant for mathematics learning and assessment contexts (Lusiyana & Juandi, 2024; Ramful et al., 2017).

Spatial visualization refers to the ability to imagine internal relationships within a shape (Mix & Cheng, 2012; Xu et al., 2025), mental rotation involves the ability to rotate objects mentally (Duffy et al., 2024; Kurt et al., 2023), and spatial orientation reflects the ability to understand the position and direction of objects from a particular perspective (Harris, 2021; Nori et al., 2023). One mathematical topic that strongly requires these abilities is geometric transformation, including translation, reflection, rotation, and dilation, which demand both visualization and mental manipulation of shapes (Harris, 2023; Hawes et al., 2017; Yuliardi & Rosjanuardi, 2021). Students with high spatial ability are generally better at performing such transformations, whereas those with low spatial ability often have difficulty understanding changes in position and orientation (Astuti et al., 2025; Febrianti et al., 2024).

Although spatial ability has been widely examined in mathematics education, several gaps remain. Previous studies have often focused on general measures of spatial ability, which may provide limited insight into how students use spatial ability in specific geometry tasks (Harris, 2023). In addition, existing research has not sufficiently connected specific aspects of spatial ability, such as spatial visualization, mental rotation, and spatial orientation, with students' cognitive

processes during problem solving (Bruce et al., 2017). This issue is important because different aspects of spatial ability may support different mathematical activities and involve different cognitive demands (Adams et al., 2023; Gilligan-Lee & Hawes, 2022). Therefore, examining students' spatial ability within geometric transformation tasks and analyzing their thinking processes through APOS theory may provide a more contextualized understanding of how spatial ability develops.

In addition to spatial ability, individual differences in information processing also influence how students solve geometric tasks. Cognitive style refers to relatively stable preferences or habitual strategies that influence how individuals perceive, learn, and solve problems (Lacko et al., 2023). Two important cognitive styles in geometry learning are Field Independent (FI) and Field Dependent (FD), which describe individual differences in the degree of dependence on surrounding visual contexts when processing information (Giancola et al., 2023; Witkin et al., 1977). FI students tend to rely more on internal frames of reference. They are better able to separate relevant information from surrounding contexts, whereas FD students tend to rely more on external frames of reference and contextual cues (Nori et al., 2023). In geometry learning, these differences may influence how students organize visual information, identify relevant geometric elements, and engage with transformation tasks (Ubuz & Aydýnyer, 2019). Several studies have reported that FI students generally demonstrate stronger spatial performance than FD students because they are better able to process information analytically, filter relevant visual information, and mentally manipulate objects (Li et al., 2023; Nori et al., 2023; Syahbudin et al., 2024). However, most of these studies focus primarily on final performance outcomes and provide limited insight into how students' spatial abilities develop, even

though understanding these processes is essential for identifying strategies, difficulties, and cognitive dynamics in learning.

To systematically examine students' thinking processes, this study employs APOS theory (Action, Process, Object, and Schema) as a conceptual framework. APOS theory views mathematical understanding as the construction of mental structures that progress from externally guided actions to internalized processes, encapsulated objects, and coherent schemas (Borji et al., 2018; Dubinsky & McDonald, 2001). This framework is appropriate for analyzing geometric transformation tasks because such tasks require students to coordinate an initial object, the transformation process, and the resulting representation, which can be traced through APOS mental constructions (Koyunkaya & Boz-Yaman, 2023). APOS theory is also relevant for comparing FI and FD students because these cognitive styles reflect different ways of selecting visual information, relying on internal or external frames of reference, and coordinating information during problem solving (Giancola et al., 2023; Nori et al., 2023). Thus, APOS analysis can reveal whether FI and FD students remain at procedural Action, develop internalized Process, encapsulate spatial transformation as Object, or coordinate their spatial understanding into Schema. Previous APOS-based studies have also found that FI and FD students may demonstrate different levels of spatial-thinking construction in geometry problems (Bintoro et al., 2021, 2022). Therefore, APOS theory provides a useful lens for identifying not only whether students produce correct spatial responses, but also how they construct, internalize, and coordinate spatial ideas across different stages of thinking.

Based on the above discussion, further investigation is needed to understand differences in the spatial abilities of students with Field Independent (FI) and Field Dependent (FD) cognitive styles across each aspect of spatial

ability, particularly in the context of two-dimensional geometric transformations. Most previous studies have focused on final spatial ability outcomes, which have not provided detailed insights into the dynamics of students' thinking stages for each spatial aspect. Therefore, this study adopts a qualitative approach based on APOS theory to explore students' spatial ability in two-dimensional geometric transformation tasks. Specifically, this study aims to: (1) analyze differences in spatial ability between FI and FD students across spatial visualization, mental rotation, and spatial orientation; (2) examine students' cognitive development based on APOS stages; and (3) identify patterns of thinking across these spatial aspects. By integrating cognitive style, specific aspects of spatial ability, and APOS theory into a single analytical framework, this study offers a process-based analysis of spatial ability. It provides a more detailed understanding of how spatial thinking develops in students with different cognitive styles.

■ METHOD

Participants

The participants in this study were six ninth-grade students from a public junior high school in Bandung, Indonesia. They were selected purposively based on their cognitive style, as determined by the Group Embedded Figures Test (GEFT). The GEFT scores range from 0 to 18, with higher scores indicating stronger field independence.

The selection of a small number of participants is consistent with a qualitative case study approach, which emphasizes depth of analysis rather than generalization. Each participant was examined in detail to explore their spatial thinking processes across different tasks and APOS stages.

Students were classified into two cognitive-style groups based on their GEFT scores. The three students with the highest scores were

categorized as Field Independent (FI), while those with the lowest scores were considered potential Field Dependent (FD) participants. To ensure the availability of analyzable qualitative data, FD participants were selected from students who attempted most of the test items and provided sufficient written responses for further interview analysis. This criterion was used to allow the researchers to examine students' thinking processes through APOS stages, rather than to select students based on higher mathematical performance. The distribution of GEFT scores and cognitive-style classifications is presented in Table 1.

Table 1. Participants' GEFT scores and cognitive style classification

Subject code	GEFT score	Category
FI-1	16	Field Independent
FI-2	17	Field Independent
FI-3	16	Field Independent
FD-1	4	Field Dependent
FD-2	4	Field Dependent
FD-3	0	Field Dependent

Research Design and Procedure

This study employed a qualitative case study approach to explore students' spatial thinking processes in solving geometry transformation tasks. The analysis was guided by the APOS (Action–Process–Object–Schema) theoretical framework (Dubinsky & McDonald, 2001), which was used to identify students' cognitive constructions during task completion. This design was chosen to obtain a detailed understanding of how students with different cognitive styles process spatial information across tasks involving spatial visualization, mental rotation, and spatial orientation.

The study was conducted in three main phases over approximately three months. First, the GEFT was administered to identify students' cognitive styles and to select participants

representing the FI and FD groups. Second, the selected students completed the spatial ability test, which consisted of geometry-transformation tasks designed to assess spatial visualization, mental rotation, and spatial orientation. Third, individual semi-structured interviews were conducted after the written test to clarify students' reasoning and mental processes, focusing on how they visualized, transformed, and oriented figures during problem solving.

All interviews were audio-recorded and transcribed verbatim. The interview transcripts were then checked through participant verification to ensure the accuracy of students' statements. The written responses and interview data were subsequently analyzed as individual cases and then compared across FI and FD groups to identify similarities and differences in students' spatial thinking processes.

Instruments

Three instruments were employed in this study: (1) the Group Embedded Figures Test (GEFT), (2) a spatial ability test, and (3) semi-structured interviews.

The GEFT was used to classify students into Field Independent (FI) and Field Dependent (FD) cognitive styles based on the field dependence–independence framework (Witkin et al., 1977). The test measures students' ability to identify simple figures embedded in complex visual patterns. It consists of 18 scored items, with scores ranging from 0 to 18. Higher scores indicate stronger field independence, whereas lower scores indicate stronger field dependence.

The spatial ability test was developed based on three aspects of spatial ability: spatial visualization, mental rotation, and spatial orientation. These aspects were connected to topics in geometric transformations at the junior high school level, including translation, rotation, and reflection. Each item was designed to represent one spatial aspect.

The test was reviewed by two mathematics education lecturers and one junior high school mathematics teacher to ensure content validity. The review focused on the relevance of each item to the intended spatial ability aspect, the alignment between items and indicators, the clarity of task instructions, and the suitability of the tasks for ninth-grade students. Based on the reviewers' suggestions, several revisions were made to

improve the clarity of task instructions, coordinate representations, and interview prompts. Construct validity was supported by mapping each item to the intended spatial aspect and by using interview data to verify students' written responses.

Semi-structured interviews were conducted after the written test to explore students' reasoning processes and verify their APOS stages during

Table 2. Spatial ability aspects and indicators

Aspect	Indicator
Spatial Visualization	Determining and drawing the final position of shapes after translation based on coordinate representations
Mental Rotation	Determining the final position of an object after rotation around a given center and direction
Spatial Orientation	Determining object position and facing direction after reflection relative to a reference line or viewpoint

problem solving. The interview questions focused on how students visualized shapes, performed transformations, explained their procedures, and interpreted spatial relationships. The interview data were used to clarify students' written responses and support the identification of their APOS stages.

Data Analysis

Data analysis was conducted using the APOS theoretical framework to identify students'

cognitive constructions across the Action, Process, Object, and Schema stages (Dubinsky & McDonald, 2001).

Students' written responses and interview transcripts were analyzed to determine their dominant APOS stage for each spatial-ability aspect: spatial visualization, mental rotation, and spatial orientation. The classification of APOS stages was based on criteria adapted from APOS theory as described by Arnon et al. (2014), as presented in Table 3.

Table 3. APOS coding criteria

APOS Stage	Coding Criteria
Action	Students perform procedures without a clear conceptual understanding.
Process	Students are able to describe the thinking processes and mental manipulations used.
Object	Students are able to explain the result as a coherent whole concept.
Schema	Students are able to generalize concepts or predict outcomes in modified situations.

In this study, APOS stages were treated as hierarchical cognitive constructions; therefore, evidence of Action was required before a student could be considered for the higher stages of

Process, Object, and Schema. To support coding consistency, each APOS stage was identified through both written responses and interview explanations. At the Action stage, students

performed explicit transformation procedures, such as drawing the initial figure or applying a transformation step, although their explanations remained procedural. At the Process stage, students were able to describe or mentally simulate the transformation process, such as imagining point displacement or object movement. At the Object stage, students explained the transformation result as a coherent whole, for example, by relating the initial object, transformation process, and final image. At the Schema stage, students were able to coordinate several relationships and generalize or predict the effects of modified transformations.

The coding process was conducted by the researcher and reviewed by the research supervisors to ensure consistency in data interpretation. Each student's response was assigned to the highest APOS stage consistently evidenced in both the written response and the interview transcript. Students' performance in each spatial aspect was determined by whether they met the corresponding indicators, based on consistency between written and verbal data.

Triangulation was conducted by comparing students' written responses with their interview explanations for each spatial ability task. When

both data sources showed consistent evidence, the corresponding spatial indicator and APOS stage were assigned. When differences arose, the researcher re-examined the written solution, the interview transcript, and the task requirements before assigning the stage. In such cases, the final classification was based on the evidence most consistently supported by the data, rather than on a single response.

■ RESULT AND DISCUSSION

The research data were obtained from the spatial ability test and follow-up interviews, which were analyzed using APOS theory. The analysis focused on differences in spatial ability between students with Field-Independent (FI) and Field-Dependent (FD) cognitive styles while solving geometric transformation tasks.

Overall, the results indicate that FI students exhibited more varied spatial ability performance and achieved higher levels of thinking than FD students. FI students tended to coordinate procedural actions with reflective mental processes. In contrast, FD students relied more heavily on visual stimuli and had difficulty mentally manipulating images of the shapes and positions of geometric objects.

Table 4. Spatial ability of FI and FD students

Student Code	Spatial Visualization	Mental Rotation	Spatial Orientation
FI-1	Achieved	Achieved	Not Achieved
FI-2	Not Achieved	Not Achieved	Not Achieved
FI-3	Achieved	Not Achieved	Achieved
FD-1	Achieved	Not Achieved	Not Achieved
FD-2	Achieved	Not Achieved	Not Achieved
FD-3	Not Achieved	Not Achieved	Not Achieved

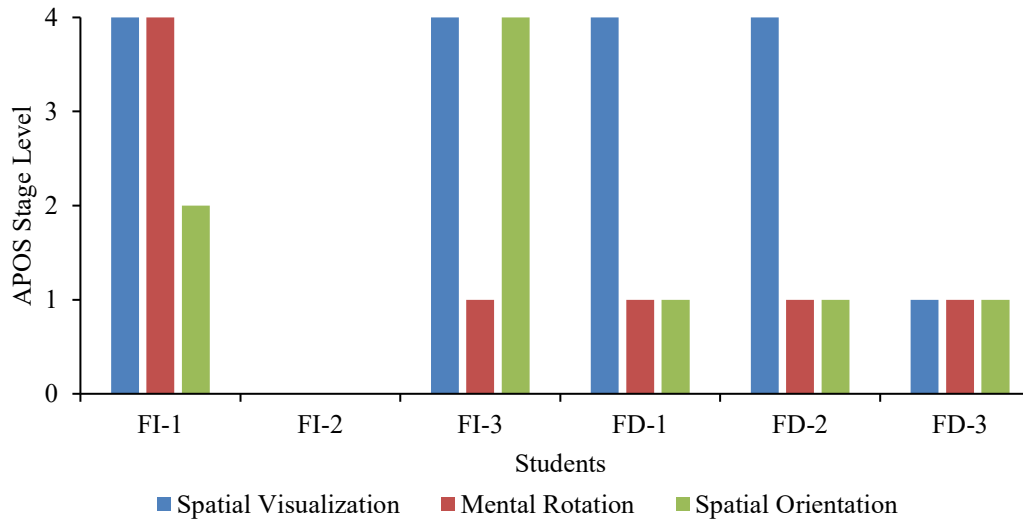
Note: Achieved indicates that the student achieved the indicator for the corresponding spatial ability aspect; Not Achieved indicates that the student did not achieve the indicator

Table 4 summarizes students' achievement across the three aspects of spatial ability, based on the indicators presented in Table 2. The table provides an overview of the distribution of

students' achievement, while the qualitative characteristics of each response are discussed in detail in the following subsections. Overall, spatial visualization was the most frequently achieved

aspect, as two FI students and two FD students achieved the indicator. In contrast, mental rotation and spatial orientation were more difficult. Only one FI student achieved the mental rotation

indicator, and only one FI student achieved the spatial orientation indicator. In contrast, none of the FD students achieved the indicators for these two aspects.



Note: APOS levels are represented as follows: 1 = Action, 2 = Process, 3 = Object, and 4 = Schema.

Figure 1. APOS stages across spatial ability aspects

Figure 1 illustrates the variation in the highest APOS stage achieved by FI and FD students across the three aspects of spatial ability. In general, FI students reached higher levels of thinking, with several attaining the Schema stage in one or more aspects. However, this pattern was not entirely consistent, as FI-2 did not demonstrate sufficient evidence to enter the Action stage across all three aspects. Meanwhile,

FD students mostly remained at the Action stage, except for two students who reached the Schema stage in the spatial visualization aspect.

Spatial Visualization

Figure 2 shows the first task used to assess students' spatial visualization skills, in which they were asked to arrange four triangles based on given coordinates to form a single closed shape.

Question 1	
Given four isosceles triangles on the coordinate plane as follows:	
-	Triangle A: (6, -2), (3, -4), (6, -6)
-	Triangle B: (-7, -5), (-1, -5), (-4, -7)
-	Triangle C: (-5, 5), (-2, 3), (-5, 1)
-	Triangle D: (1, 5), (3, 2), (1, -1)
a.	Draw the four triangles on the coordinate plane.
b.	Determine the plane figure formed if triangles B, C, and D are moved to attach to triangle A so that they form a single closed shape (e.g., a square, rectangle, trapezoid, or another closed shape).
c.	Draw the final closed shape that is formed.
d.	Explain the transformation steps (translation, rotation, or a combination of both) that you used to construct the shape.

Figure 2. Spatial visualization task

This task aimed to measure students' ability to visualize spatial relationships and organize shapes coherently based on given spatial information.

Figure 3 presents examples of FI students' responses in completing the spatial visualization task. Two FI students (FI-1 and FI-3)

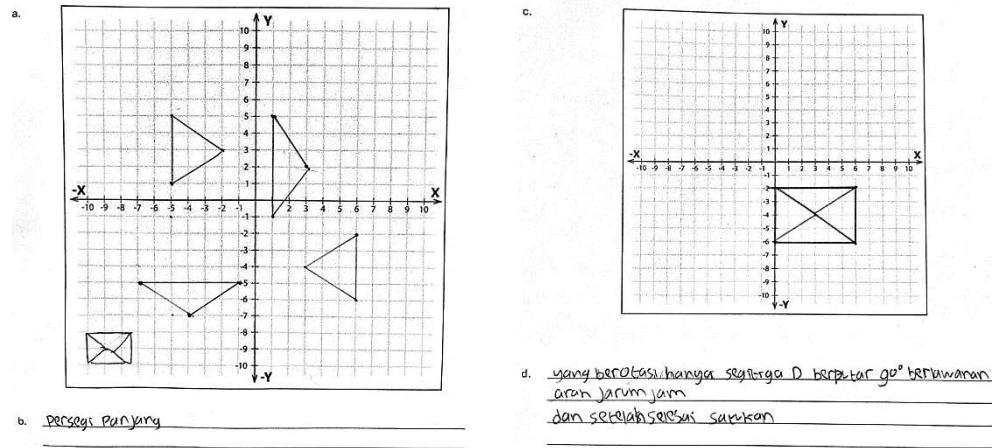


Figure 3. Example of FI student's response (FI-3)

successfully arranged the four triangles into a rectangle as required by the task, demonstrating their ability to visualize relationships among components and organize shapes into a coherent whole. These students progressed to the Schema stage, as they were able to generalize the relationships among the triangles and understand that changes in one part affected the overall shape's balance.

However, FI-2 did not demonstrate sufficient evidence to be classified at the Action

stage in the spatial visualization task because he was unable to correctly identify the coordinates and construct the basic shapes required in the task. This difficulty was also reflected in his interview response: "I did not understand, Sir. I was also confused by the problem." As a result, the subsequent stages: Process, Object, and Schema, could not be analyzed.

Figure 4 shows examples of FD students' responses to the same spatial visualization task. Two FD students (FD-1 and FD-2) were able

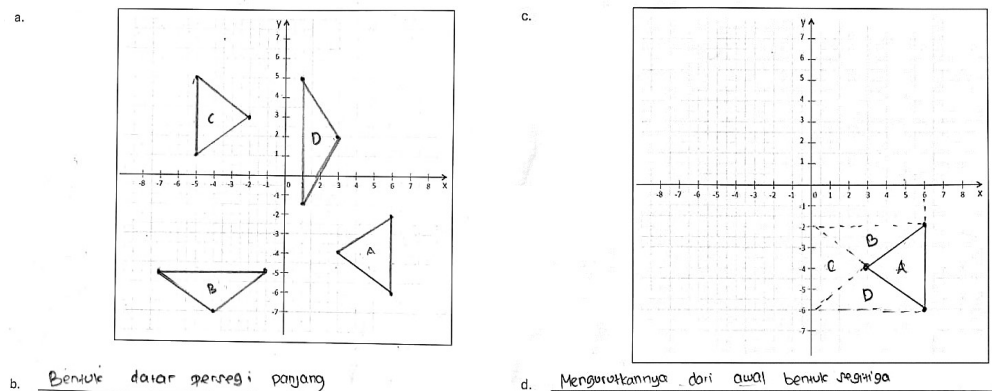


Figure 4. Example of FD student's response (FD-1)

to construct the rectangular shape correctly and reached the Schema stage in this specific task. They organized the given triangles into a coherent

arrangement using the visual and coordinate-based information provided in the task. One FD student (FD-3), however, reached only the

Action stage because he did not adequately consider the triangles' size and orientation, resulting in a final shape that did not match the task instructions.

In the spatial visualization aspect, FI students generally demonstrated stronger performance than in the other spatial aspects examined in this study. This may be because the task allowed students to use visual information directly to organize relationships among shape components, without requiring complex mental manipulation or a change in perspective. This interpretation is consistent with the view that spatial visualization supports students in generating, maintaining, and organizing visual-spatial information in geometry tasks (Hawes et al., 2017; Nadzeri et al., 2024). FI-1 and FI-3 were able to identify relevant visual elements, imagine the configuration of the four triangles, and arrange them into a coherent whole. This finding is also supported by Duffy et al. (2024), who emphasized the role of spatial ability in the formation of structural mental representations, and by Li et al. (2023), who noted that FI students tend to filter relevant information more effectively.

However, the case of FI-2 shows that FI cognitive style alone does not guarantee successful spatial visualization. From the perspective of APOS theory, FI-2's difficulty occurred before the Action stage because he had not established the initial external actions needed to begin the problem-solving process. Although FI students are generally expected to process information analytically and independently, FI-2 showed difficulty in identifying coordinates, constructing initial figures, and applying transformation procedures. Thus, his failure appears to be related to weak prerequisite conceptual and procedural knowledge, particularly in coordinate interpretation and basic figure construction, rather than to cognitive style alone. This case highlights that prerequisite understanding may mediate the relationship

between cognitive style and spatial ability (Borji & Martínez-Planell, 2020).

In contrast, some FD students were also able to achieve the spatial visualization indicator by correctly drawing the initial shapes and arranging the components based on clear visual cues. This finding is consistent with the concept of spatial visualization, which emphasizes the ability to organize internal relationships within a shape (Xu et al., 2025; Yuliardi & Rosjanuardi, 2021). The Schema-level achievement of FD-1 and FD-2 can be understood in relation to the characteristics of the task, which provided explicit visual and coordinate-based cues that supported their organization of the given shapes. This interpretation is consistent with Ubuz & Aydınyer (2019), who noted that FD students tend to rely on salient contextual cues and may benefit from visual and concrete representations in geometry learning. However, their success should still be interpreted with caution because it emerged on a task that provided visible components and a structured arrangement process, rather than on a task requiring fully independent mental manipulation.

From the perspective of APOS theory, FI students generally showed more complex thinking development than FD students. Two FI students reached the Schema stage by recognizing relational patterns among shape components and generalizing them to other contexts, which reflects the coordination of actions, processes, and objects into a coherent cognitive structure (Borji et al., 2018). However, this pattern was not absolute, as one FI student did not demonstrate sufficient evidence to be classified at the Action stage due to difficulty in establishing the initial mathematical actions required in the task. Meanwhile, among FD students, the Schema stage appeared in a few cases, particularly when the task provided clear visual and coordinate-based support. This finding suggests that although FD students were sometimes able to produce

correct visualization results, their conceptual construction was still strongly supported by concrete visual cues. Overall, FI students were more likely to develop coherent APOS constructions in spatial tasks, although the development of APOS stages was also influenced by task structure, visual support, and students'

prerequisite understanding (Bintoro et al., 2022; Wulandari et al., 2019).

Mental Rotation

Figure 5 shows the second task that assessed students' mental rotation ability. In this task, students were asked to rotate a given figure

Question 2
Given an L-shaped figure with vertices at (1, 1), (3, 1), (3, 2), (2, 2), (2, 4), and (1, 4). a. Draw the L-shaped figure on the coordinate plane. b. Rotate the figure 90° counterclockwise about the center point (1, -1), then write the coordinates of the image. c. Rotate the image from (b) 180° about the center point (-2, 2), then write the coordinates of the resulting image. d. Draw the final figure obtained after performing the rotations.

Figure 5. Mental rotation task

by the specified direction and degree, and then determine the new coordinates of the resulting image. This task required students to mentally manipulate the figure's position to achieve the correct orientation.

Figure 6 presents examples of FI students' responses to the mental rotation task. One FI student (FI-1) successfully drew the L-shaped figure and performed both rotations as instructed: a 90° counterclockwise rotation followed by a

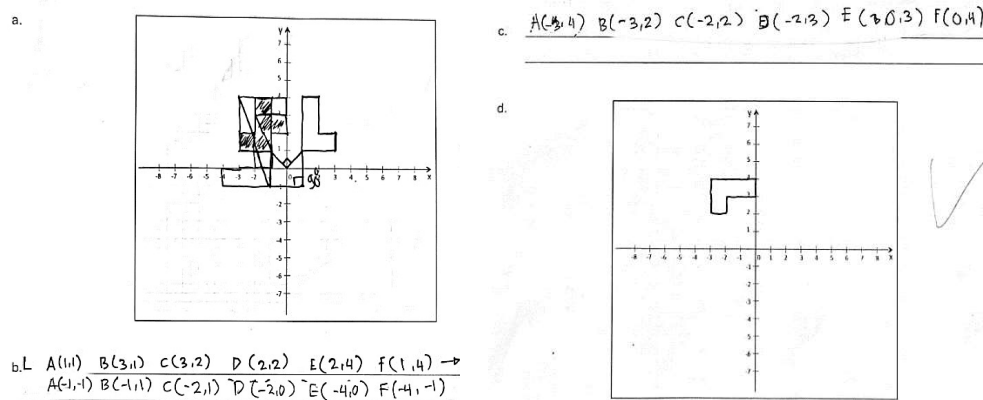


Figure 6. Example of FI student's response (FI-1)

180° rotation about a given center. The direction of rotation and the final position of the shape were presented correctly and were consistent with the resulting coordinates. This indicates that the indicators of mental rotation ability were fulfilled.

Based on the interview, FI-1 appeared to visualize the sequence of rotations without solely

redrawing the shape, as he recognized that changing the order of rotations would yield a different final position. This was reflected in his statement: "It should be different. If the 180° rotation is done first and then the 90° counterclockwise rotation, the position will be different from the first result." This response

indicates that FI-1 did not merely apply a procedural rule but also coordinated the sequence of rotations and recognized that the order of transformations affects the final position. Therefore, FI-1 was classified at the Schema stage because he was able to coordinate the rotation sequence, reference points, and final position as an integrated transformation process.

In contrast, FI-2 could not be categorized at the Action stage because he did not draw the rotated shape or write the coordinates after the

rotation. He only marked the initial points, indicating that the Action stage had not been achieved; therefore, the subsequent stages (Process, Object, and Schema) could not be analyzed. FI-3 was able to draw the results of two rotations but incorrectly identified the center of the first rotation, resulting in an inaccurate final position. This error indicates that FI-3 reached only the Action stage, as he had not yet conceptually understood the relationship between the rotation center and the direction of rotation.

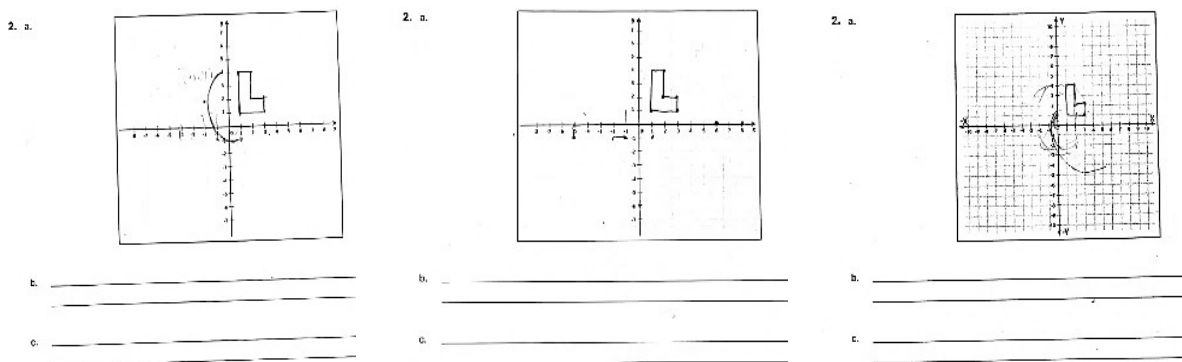


Figure 7. Examples of FD students' responses (FD-1, FD-2, and FD-3)

Figure 7 shows the FD students' responses to the mental rotation task. All FD students (Figures 6a, 6b, and 6c) drew only the initial shape without performing the required rotations. Although they correctly determined the initial coordinates, they did not show any positional changes due to rotation. This indicates that they were unable to mentally rotate the object and therefore did not achieve the indicators of mental rotation ability. The interview results further showed that none of the FD students could explain the direction of rotation, the distance from the center, or the shape's final position. Consequently, all FD students remained at the Action stage and did not progress to the Process or Schema stages.

In the mental rotation aspect, the performance gap between FI and FD students became more evident than in spatial visualization. Only FI-1 fulfilled the mental rotation indicator, whereas FI-2 and FI-3 did not. FI-2 was unable to initiate the rotation procedure, while FI-3

attempted the rotation but incorrectly identified the rotation center, resulting in an inaccurate final position. This finding suggests that FI cognitive style alone does not guarantee successful mental rotation when students have not developed sufficient conceptual understanding of geometric transformation. Although FI students tend to demonstrate systematic and analytical information processing (Li et al., 2023), mental rotation requires more than analytical attention to visual elements. It involves the continuous, dynamic manipulation of mental images, including coordinating the direction, angle, center of rotation, and resulting position, thereby making it cognitively more demanding than spatial visualization (Harris, 2023; Rutherford et al., 2018; Schenck & Nathan, 2024).

All FD students failed to achieve the mental rotation indicator. Although they were able to draw the initial shape correctly, they did not continue the rotation, making it impossible to determine

the shape's final position. This suggests that the visual information available in the task supported FD students only at the initial representation stage but was insufficient to perform the internal mental manipulation required for rotation. Mental rotation requires students to mentally transform object images by coordinating the direction, angle, center of rotation, and resulting position, making it one of the more demanding spatial processes (Kurt et al., 2023; Rutherford et al., 2018). Therefore, FD students' difficulty in this task may be related to their reliance on visible cues and limited internal spatial coordination when the task required them to manipulate an object that was no longer directly observable. From the APOS perspective, this reliance on external visual cues may limit the internalization required to progress from Action to Process, as students rely on directly observable representations rather than mentally coordinating transformation relationships. These difficulties were also influenced by a limited understanding of transformation concepts and the task's high cognitive demands (Hidayati et al., 2025; Lowrie & Logan, 2023; Sabil et al., 2024).

From the perspective of APOS theory, the differences in thinking stages between FI and FD students were clearly visible. Among the FI students, FI-1 reached the Schema stage because he was able to coordinate the sequence of rotations and explain that changing the order of

rotation would produce a different final position. His interview response also indicated awareness that changes in the reference points could affect the resulting position, reflecting reflective thinking in integrating multiple spatial processes (Hasdi et al., 2024). FI-2 showed no evidence of the initial action required in the task because he did not initiate the required rotation procedure. At the same time, FI-3 remained at the Action stage because he performed part of the procedure but incorrectly identified the rotation center. In contrast, all FD students remained at the Action stage because they were able to construct the initial figure but did not demonstrate the mental image manipulation required to complete the rotational transformation. This finding suggests that progression beyond the Action stage in mental rotation requires students to coordinate the direction, angle, rotation center, and final position as an integrated transformation process (Borji et al., 2018; Harris, 2023). Such coordination is particularly demanding for FD students because spatial transformation tasks require high-level mental coordination and are more difficult when students rely strongly on external visual cues (Nori et al., 2023; Xu et al., 2025).

Spatial Orientation

Figure 8 shows the third task, which assessed spatial orientation by asking students

Question 3
A student is located at point (4, 2) and is facing a house on the coordinate plane with the following vertices: (3, 3), (5, 3), (5, 4), (6, 4), (4, 5), (2, 4), and (3, 4). Also shown on the coordinate plane are: - Triangle: (-1, 1), (-3, 1), (-2, 2) - Tree : (4, -4) a. Draw the house, the triangle, the student (S), and the tree (P) on the coordinate plane, then reflect the house and the student across the line $y = -x$. b. After reflecting across the line $y = -x$, determine the position of the tree relative to the student (choose one: left, right, in front of, or behind). c. After reflecting across the line $y = -x$, determine the position of the triangle relative to the student (choose one: left, right, in front of, or behind).

Figure 8. Spatial orientation task

to determine the position and facing direction of objects after a reflection across a line. This task required students to interpret how each object's position and orientation change when reflected over an oblique mirror line.

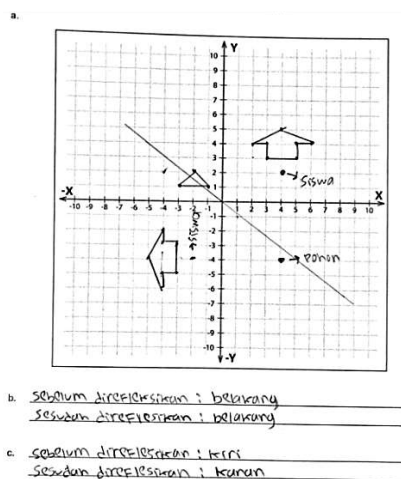


Figure 9. Example of FI student's response (FI-3)

Figure 9 presents examples of FI students' responses in completing the spatial-orientation task. One FI student (FI-3) correctly presented the reflection of the house-shaped figure across the line $y = -x$. He was able to determine the position of the house, the facing direction of character S, and the positional changes of tree P and the triangle after the transformation, indicating that the spatial orientation indicator was achieved. Based on the interview, FI-3 used a paper-folding analogy to visualize the reflection process, as shown in his statement: *"I folded the paper, Sir, so the direction of the house goes here [pointing to the correct position], and then I just matched the points when it was folded."* This response indicates that FI-3 had reached the Process stage because he could mentally simulate the reflection process through the paper-folding analogy. He was then classified at the Schema stage because his subsequent explanation showed that he could generalize the relationship between the mirror line, point

displacement, and changes in object orientation. The use of the paper-folding analogy also suggests that FI students' spatial reasoning should not be understood as purely abstract, because concrete representation may function as a bridge for mentally restructuring spatial relations in a demanding orientation task.

FI-1 was also able to determine the reflected position of the figure; however, he was not accurate in identifying the final orientation or facing direction of character S relative to other objects. He reached the Process stage because he was able to mentally imagine point displacement, but did not progress to the Object or Schema stages due to inconsistency in the final orientation. Meanwhile, FI-2 failed to present the reflected image and the facing direction of the objects; he incorrectly drew the initial figure and did not perform the transformation, indicating that even the Action stage was not achieved.

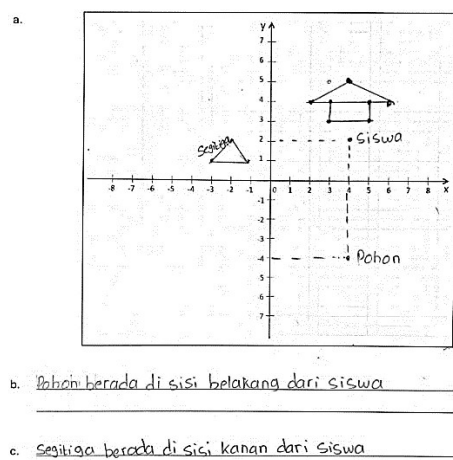


Figure 10. Example of FD student's response (FD-2)

Figure 10 shows an example of FD students' responses to the spatial-orientation task. All FD students (FD-1, FD-2, and FD-3) failed to present reflections across the line $y = -x$. They drew only the initial figure, without showing the reflected image or any changes in object orientation. Interview data revealed that they

experienced confusion when dealing with a slanted mirror line and did not understand the reflection procedure. This indicates that the indicators of spatial orientation ability were not achieved. Although all FD students were able to draw the initial figure according to the given coordinates, they remained at the Action stage because they did not demonstrate any mental processes or conceptual understanding of changes in object orientation.

FI students also demonstrated limited achievement, as only one student achieved the established indicator. Theoretically, FI students are expected to perform better in this aspect because field independence supports spatial updating and the internal adjustment of viewing direction and object position (Harris, 2021; Nori et al., 2023). However, difficulties appeared when the task required students to determine changes in facing direction after reflection across a slanted line. The use of a paper-folding analogy by FI-3 suggests that even FI students may rely on concrete representations to support abstract spatial reasoning when the task becomes cognitively demanding. This indicates that spatial orientation does not depend solely on visual observation, but also requires conceptual understanding of positional and perspectival changes at a mental level. Such difficulty is consistent with the view that perspective-taking requires learners to move beyond their own viewpoint and imagine spatial relations from another position (Bruce et al., 2017; Harris, 2023). As a result, spatial orientation imposes a considerable cognitive on students, particularly when the task requires them to coordinate object position, mirror-line direction, and orientation changes simultaneously (Xu et al., 2025).

All FD students failed to achieve the spatial orientation indicator. They only drew the initial figure without presenting the reflected image across the line $y = -x$ or indicating the change in the character's facing direction after the

transformation. This suggests that the available visual information supported FD students only at the initial representation stage but was insufficient to determine relative positions and orientation from a new perspective. Their difficulty was further reflected in their confusion about reflection procedures involving a slanted mirror line, indicating that both conceptual understanding of reflection and the ability to mentally shift perspective had not yet developed. This result is consistent with the view that spatial orientation and perspective-taking require learners to move beyond immediate visual information and imagine spatial relations from a different viewpoint (Bruce et al., 2017; Harris, 2023).

From the perspective of APOS theory, clear differences in thinking stages between FI and FD students were observed in the spatial orientation task. Among the FI students, FI-2 showed no evidence of the initial action required in the task because he was unable to correctly construct the initial figure and did not perform the reflection process. FI-1 reached the Process stage because he was able to imagine point displacement, but his understanding was not yet stable enough to consistently determine the final orientation or facing direction. Meanwhile, FI-3 reached the Schema stage because he was able to connect the position of the mirror line, point displacement, and changes in object orientation into a coherent reflection structure. This ability reflects reflective thinking in coordinating multiple spatial processes (Hasdi et al., 2024). It is consistent with the APOS view that higher understanding develops through the coordination of actions, processes, and objects into a coherent schema (Borji et al., 2018). In contrast, all FD students remained at the Action stage because they were able to draw the initial figure but did not demonstrate the mental image manipulation required to update direction and spatial position after reflection. This suggests that their dependence on directly observable representations may have limited the

internalization of perspective changes and the mental coordination of object orientation after reflection (Harris, 2021; Nori et al., 2023; Xu et al., 2025).

To synthesize the cross-case findings, Table 5 summarizes the APOS-based cognitive profiles of FI and FD students across the three aspects of spatial ability.

Table 5. APOS-Based cognitive profile

Student	APOS-Based Cognitive Profile
FI-1	FI-1 constructed Schema-level thinking in spatial visualization and mental rotation, but his spatial orientation remained at the Process stage because object position and facing direction were not fully coordinated.
FI-2	FI-2 did not establish the initial Action construction needed to interpret coordinates and initiate transformations across the three spatial aspects.
FI-3	FI-3 constructed a schema when visual relations and reflection structures could be coordinated, but his mental rotation was constrained by a misconception about the rotation center.
FD-1	FD-1 reached Schema only in cue-supported visualization, while tasks requiring internal transformation remained at Action.
FD-2	FD-2 organized visible components into a coherent visualization Schema, but rotation and orientation processes were not interiorized.
FD-3	FD-3 remained at Action because his responses showed initial representation without mental transformation.

This synthesis shows that FI students generally demonstrated more advanced APOS constructions, particularly when tasks required coordinating multiple spatial relationships. However, this pattern was not entirely consistent because prerequisite understanding also influenced students' progression across APOS stages. In contrast, FD students tended to rely more heavily on visible representations and external cues, which supported visualization tasks but limited the internalization processes required for mental rotation and spatial orientation.

Cross-Aspect Analysis of FI-2

Although FI students generally demonstrated higher spatial achievement, FI-2 showed a contrasting pattern, as he did not provide sufficient evidence to reach the Action stage across all three spatial ability aspects. In the spatial visualization task, FI-2 was unable to correctly identify the coordinates and construct the basic shapes. This difficulty was reflected in his interview response: "I did not understand, Sir.

I was also confused by the problem." Similar difficulties appeared in the mental rotation task, where he marked only the initial points, without drawing the rotated figure or determining the resulting coordinates. In the spatial orientation task, FI-2 also failed to reconstruct the initial figure accurately and did not proceed to the reflection process.

This cross-aspect pattern suggests that FI-2's difficulty was not simply related to cognitive style, but was more strongly associated with weak prerequisite conceptual and procedural understanding, particularly in interpreting coordinates and initiating transformation procedures. From the APOS perspective, FI-2 did not demonstrate the initial external actions required to begin the problem-solving process; therefore, the higher stages of Process, Object, and Schema could not be analyzed. This case indicates that a field-independent cognitive style may support spatial reasoning only when students have sufficient foundational knowledge to meet the task demands.

However, these findings should be interpreted with caution due to several limitations of the study. This research involved only six ninth-grade students from one junior high school. Hence, the findings mainly reflect the characteristics of the selected participants and cannot be generalized to a wider student population. The small number of participants may also limit the transferability of the findings, because students' spatial ability and APOS thinking stages may vary across different school contexts, learning experiences, and levels of prior knowledge. In terms of credibility, this study sought to strengthen the trustworthiness of the findings by triangulating written responses and interview data. Nevertheless, the limited number of cases means that the identified patterns should be viewed as context-specific rather than as stable patterns representing all FI and FD students. Future studies could involve more participants from different schools and examine spatial ability in other mathematical topics to obtain broader and more transferable insights.

■ CONCLUSION

The findings of this study show that students' spatial ability in geometric transformation tasks varied across cognitive styles, spatial aspects, and APOS thinking stages. FI students generally demonstrated higher and more diverse achievement, particularly in spatial visualization, although their performance in mental rotation and spatial orientation remained limited and inconsistent. In contrast, FD students showed achievement mainly in spatial visualization and experienced substantial difficulties in mental rotation and spatial orientation. The APOS analysis further revealed that FI students tended to reach higher levels of thinking, including the Schema stage. In contrast, most FD students remained at the Action stage, except in spatial visualization tasks, where clear visual and

coordinate-based support enabled some FD students to reach the Schema stage. These findings are important for mathematics education because they show that spatial ability in geometry learning is determined not only by cognitive style but also by task demands, visual support, and students' prerequisite understanding. In addition, APOS theory provides a useful framework for identifying how cognitive style, visual support, and prerequisite understanding interact in the development of students' spatial reasoning.

The results imply that geometry instruction should explicitly support students in developing mental image manipulation skills, understanding transformation procedures, and coordinating changes in spatial perspective. Teachers need to provide structured visual support, concrete representations, and gradual transitions toward more independent mental manipulation, especially for tasks involving rotation and spatial orientation. At the same time, the findings should be interpreted cautiously because this study involved only six ninth-grade students from one junior high school and focused specifically on geometric transformation tasks within the APOS framework. Therefore, the results are context-specific and cannot be generalized broadly. Future studies should involve more participants from diverse school contexts and examine students' spatial ability across other geometry topics to yield broader, more transferable insights.

■ DECLARATION OF GENERATIVE AI USAGE IN THE WRITING PROCESS

During the drafting and revision of this manuscript, the authors used ChatGPT to refine sentence structure, improve academic language, and assist with translating several sections of the text. Following the use of this tool, the authors reviewed and revised the content as necessary and accept full responsibility for the final content of the article.

■ REFERENCES

- Adams, J., Resnick, I., & Lowrie, T. (2023). Supporting senior high-school students' measurement and geometry performance: Does spatial training transfer to mathematics achievement?. *Mathematics Education Research Journal*, 35(4), 879–900. <https://doi.org/10.1007/s13394-022-00416-y>
- Arnon, I., Cottrill, J., Dubinsky, E., Oktaç, A., Fuentes, S. R., Trigueros, M., & Weller, K. (2013). Mental structures and mechanisms: APOS theory and the construction of mathematical knowledge. In *APOS theory: A framework for research and curriculum development in mathematics education* (pp. 17–26). New York, NY: Springer New York.
- Astuti, E. P., Reza, S. H., Ngulia, K. N., Lestari, D., Saktiyani, D., & Pradana, S. L. (2025). Analysis of spatial reasoning ability in solving geometric transformation problems. *Jurnal Math Educator Nusantara*, 11(2), 279–291. <https://doi.org/10.29407/jmen.v11i2.27277>
- Bintoro, H. S., Sukestiyarno, Y. L., Mulyono, & Walid. (2021). The spatial thinking process of the field-independent students based on action–process–object–schema theory. *European Journal of Educational Research*, 10(4), 1807–1823. <https://doi.org/10.12973/EU-JER.10.4.1807>
- Bintoro, H. S., Sukestiyarno, Y. L., Mulyono, M., & Walid, W. (2022). The spatial thinking process of the field-dependent students in reconstructing the geometrical concept. *International Journal of Evaluation and Research in Education*, 11(3), 1116–1124. <https://doi.org/10.11591/ijere.v11i3.22399>
- Borji, V., Alamolhodaei, H., & Radmehr, F. (2018). Application of the APOS-ACE theory to improve students' graphical understanding of derivative. *EURASIA Journal of Mathematics, Science and Technology Education*, 14(7), 2947–2967. <https://doi.org/10.29333/ejmste/91451>
- Borji, V., & Martínez-Planell, R. (2020). On students' understanding of implicit differentiation based on APOS theory. *Educational Studies in Mathematics*, 105(2), 163–179. <https://doi.org/10.1007/s10649-020-09991-y>
- Bruce, C. D., Davis, B., Sinclair, N., McGarvey, L., Hallowell, D., Drefs, M., Francis, K., Hawes, Z., Moss, J., Mulligan, J., & Okamoto, Y. (2017). Understanding gaps in research networks: Using “spatial reasoning” as a window into the importance of networked educational research. *Educational Studies in Mathematics*, 95, 143–161. <https://doi.org/10.1007/s10649-016-9743-2>
- Dubinsky, E., & McDonald, M. A. (2001). APOS: A constructivist theory of learning in undergraduate mathematics education research. In D. Holton, M. Artigue, U. Kirchgräber, J. Hillel, M. Niss, & A. Schoenfeld (Eds.), *The teaching and learning of mathematics at university level: An ICMI study* (Vol. 7, pp. 275–282). Springer Netherlands. https://doi.org/10.1007/0-306-47231-7_25
- Duffy, G., Sorby, S., & Bowe, B. (2024). Exploring the role of spatial ability in the mental representation of word problems in mathematics. *Frontiers in Education*, 9, Article 1346474. <https://doi.org/10.3389/educ.2024.1346474>
- Febrianti, D. A., Emmanuela, E., Tambunan, B., & Tarigan, G. H. (2024). Peningkatan kemampuan spasial siswa SMP Negeri 17 kelas IX melalui penerapan RME berbantuan media interaktif GeoGebra pada materi transformasi geometri

- [Improving the spatial abilities of grade IX students of SMP Negeri 17 through the application of RME assisted by interactive GeoGebra media on geometric transformation material.]. *JagoMIPA: Jurnal Pendidikan Matematika dan IPA*, 4(3), 553–561. <https://doi.org/10.53299/jagomipa.v4i3.792>
- Giancola, M., D'Amico, S., & Palmiero, M. (2023). Working memory and divergent thinking: The moderating role of field-dependent-independent cognitive style in adolescence. *Behavioral Sciences*, 13(5), Article 397. <https://doi.org/10.3390/bs13050397>
- Gilligan-Lee, K. A., & Hawes, Z. C. K. (2022). Spatial thinking as the missing piece in mathematics curricula. *npj Science of Learning*, 7, Article 10. <https://doi.org/10.1038/s41539-022-00128-9>
- Harris, D. (2021). Spatial ability, skills, reasoning or thinking: What does it mean for mathematics? In Y. H. Leong et al. (Eds.), *Excellence in mathematics education: Foundations and pathways (Proceedings of the 43rd Annual Conference of the Mathematics Education Research Group of Australasia)*, pp. 219–226. MERGA.
- Harris, D. (2023). Spatial reasoning in context: Bridging cognitive and educational perspectives of spatial-mathematics relations. *Frontiers in Education*, 8, Article 1302099. <https://doi.org/10.3389/educ.2023.1302099>
- Hasdi, Manuharawati, & Sulaiman, R. (2024). *Kemampuan pemecahan masalah matematika siswa sekolah dasar dengan gaya kognitif field dependent* [Mathematical problem-solving abilities of elementary school students with field-dependent cognitive style]. *EDUKASIA: Jurnal Pendidikan dan Pembelajaran*, 5(1), 1431–1438. <https://doi.org/10.62775/edukasia.v5i1.1040>
- Hawes, Z., Moss, J., Caswell, B., Naqvi, S., & MacKinnon, S. (2017). Enhancing children's spatial and numerical skills through a dynamic spatial approach to early geometry instruction: Effects of a 32-week intervention. *Cognition and Instruction*, 35(3), 236–264. <https://doi.org/10.1080/07370008.2017.1323902>
- Hidayati, A., Rusijono, R., Arianto, F., Prasetya, S. P., & Murtini, S. (2025). The influence of the map-assisted problem-based learning model and cognitive style on the spatial and critical thinking abilities of junior high school students. *Edelweiss Applied Science and Technology*, 9(9), 1968–1983. <https://doi.org/10.55214/2576-8484.v9i9.10281>
- Koyunkaya, M. Y., & Boz-Yaman, B. (2023). Changes in students' mental constructions of function transformations through the APOS framework. *International Electronic Journal of Mathematics Education*, 18(4), Article em0747. <https://doi.org/10.29333/iejme/13515>
- Kurt, G., Önel, F., & Çakýođlu, Ö. (2023). An investigation of middle school students' spatial reasoning skills. *International Electronic Journal of Elementary Education*, 16(1), 123–141. <https://doi.org/10.26822/iejee.2023.319>
- Lacko, D., Prošek, T., Èenik, J., Helísková, M., Ugwitz, P., Svoboda, V., Poèaji, P., Vais, M., Halíøová, H., Juøík, V., & Šašinka, È. (2023). Analytic and holistic cognitive style as a set of independent manifests: Evidence from a validation study of six measurement instruments. *PLoS ONE*, 18(6), Article e0287057. <https://doi.org/10.1371/journal.pone.0287057>
- Li, C., Mu, X., Tan, Y., Gu, C., Hu, B. Y., &

- Fan, C. (2023). Do field-dependent individuals tend to have lower creativity than field-independent ones? The role of informational cues in electronic brainstorming. *Interactive Learning Environments*, 31(2), 1106–1125. <https://doi.org/10.1080/10494820.2020.1821715>
- Lowrie, T., & Logan, T. (2023). Spatial visualization supports students' math: Mechanisms for spatial transfer. *Journal of Intelligence*, 11(6), Article 127. <https://doi.org/10.3390/jintelligence11060127>
- Lusiñana, D., & Juandi, D. (2024). Analysis of spatial ability in mathematics learning: A systematic literature review. *Lentera Sriwijaya: Jurnal Ilmiah Pendidikan Matematika*, 6(1), 28–36. <https://doi.org/10.36706/jls.v6i1.2>
- Maier, P. H. (1996). Spatial geometry and spatial ability: How to make solid geometry solid? In K. P. Müller (Ed.), *Beiträge zum Mathematikunterricht 1996: Vorträge auf der 30. Tagung für Didaktik der Mathematik* (pp. 272–275). Franzbecker
- Maresch, G. (2013). Spatial ability: The phases of spatial ability research. *Journal for Geometry and Graphics*, 17(2), 237–250.
- Mix, K. S., & Cheng, Y. L. (2012). The relation between space and math: Developmental and educational implications. In J. B. Benson (Ed.), *Advances in child development and behavior*, 42, 197–243. Elsevier. <https://doi.org/10.1016/B978-0-12-394388-0.00006-X>
- Nadzeri, Musa, M., Meng, C. C., & Ismail, I. M. (2024). The effects of augmented reality geometry learning applications on spatial visualization ability for lower primary school pupils. *International Journal of Interactive Mobile Technologies*, 18(16), 104–118. <https://doi.org/10.3991/ijim.v18i16.47079>
- Newcombe, N. S. (2016). Thinking spatially in the science classroom. *Current Opinion in Behavioral Sciences*, 10, 1–6. <https://doi.org/10.1016/j.cobeha.2016.04.010>
- Newcombe, N. S., & Frick, A. (2010). Early education for spatial intelligence: Why, what, and how. *Mind, Brain, and Education*, 4(3), 102–111. <https://doi.org/10.1111/j.1751-228X.2010.01089.x>
- Nori, R., Boccia, M., Palmiero, M., & Piccardi, L. (2023). The contribution of field independence in virtual spatial updating. *Current Psychology*, 42(6), 4567–4576. <https://doi.org/10.1007/s12144-021-01788-3>
- Ramful, A., Lowrie, T., & Logan, T. (2017). Measurement of spatial ability: Construction and validation of the spatial reasoning instrument for middle school students. *Journal of Psychoeducational Assessment*, 35(7), 709–727. <https://doi.org/10.1177/0734282916659207>
- Rutherford, T., Kessler, S., & Lee, D. S. (2018). Is the spatial/math connection unique? Associations between mental rotation and elementary mathematics and English achievement. *Learning and Individual Differences*, 62, 180–199. <https://doi.org/10.1016/j.lindif.2018.01.014>
- Sabil, H., Simanjuntak, S. M. O. U., Iriani, D., & Junita, R. (2024). Analysis of students' spatial ability in geometry material. *JPI (Jurnal Pendidikan Indonesia)*, 13(3), 436–448. <https://doi.org/10.23887/jpiundiksha.v13i3.77083>
- Schenck, K. E., & Nathan, M. J. (2024). Navigating spatial ability for mathematics education: A review and roadmap. *Educational Psychology Review*, 36(3), Article 89. <https://doi.org/10.1007/s10648-024-09935-5>
- Syahbudin, F., Priatna, N., & Yulianti, K. (2024). Analyzing cognitive styles and spatial abilities in geometry transformations

- through the problem-based learning approach. *Jurnal Pendidikan Progresif*, 14(3), 2130–2144. <https://doi.org/10.23960/jpp.v14.i3.2024144>
- Ubuz, B., & Aydınyer, Y. (2019). Project-based geometry learning: Knowledge and attitude of field-dependent/independent cognitive style students. *Journal of Educational Research*, 112(3), 285–300. <https://doi.org/10.1080/00220671.2018.1502138>
- Wai, J., Lubinski, D., & Benbow, C. P. (2009). Spatial ability for STEM domains: Aligning over 50 years of cumulative psychological knowledge solidifies its importance. *Journal of Educational Psychology*, 101(4), 817–835. <https://doi.org/10.1037/a0016127>
- Witkin, H. A., Moore, C. A., Goodenough, D. R., & Cox, P. W. (1977). Field-dependent and field-independent cognitive styles and their educational implications. *Review of Educational Research*, 47(1), 1–64. <https://doi.org/10.3102/00346543047001001>
- Wulandari, E. D., Hidayanto, E., Subanji, & Rahardi, R. (2019). Mathematical representation of cerebral palsy students in constructing the concept of plane geometry based on APOS theory. *Journal of Physics: Conference Series*, 1227(1), Article 012018. <https://doi.org/10.1088/1742-6596/1227/1/012018>
- Xu, T., Sun, S., & Kong, Q. (2025). Spatial reasoning and its contribution to mathematical performance across different content domains: Evidence from Chinese students. *Journal of Intelligence*, 13(4), Article 41. <https://doi.org/10.3390/jintelligence13040041>
- Yıldızhan, B., & Ertekin, E. (2024). A bibliometric review on spatial ability studies in education. *Educational Academic Research*, 1–20. <https://doi.org/10.33418/education.1421882>
- Yuliardi, R., & Rosjanuardi, R. (2021). Hypothetical learning trajectory in student's spatial abilities to learn geometric transformation. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, 6(3), 174–190. <https://doi.org/10.23917/jramathedu.v6i3.13338>