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The Role of Ethnomathematical Context in Geometric Reasoning: A Van Hiele-Based Analysis of Indonesian Eighth-Graders

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Abstract: The Role of Ethnomathematical Context in Geometric Reasoning: A Van Hiele-Based Analysis of Indonesian Seventh-Graders. **Objective**: This study aims to analyze students' geometric thinking skills on plane geometry material based on Van Hiele's theory. An ethnomathematics approach was employed, focusing on the geometric patterns and shapes found in the ornaments, windows, and domes of local mosques, which serve as representations of plane geometry. This study is intended to describe the profile of students' thinking levels and the difficulties that arise at each stage of geometric thinking, without testing the improvement or influence of specific interventions. **Methods**: The study employed a qualitative descriptive approach with 22 eighth-grader students from a public junior high school in Majalengka Regency, Indonesia, as participants. The instruments used included a diagnostic test based on Van Hiele's theory and unstructured interviews to explore students' reasoning patterns. Data were analyzed using thematic analysis through the stages of reduction, presentation, and conclusion drawing, and validated through triangulation between test and interview results. Classification of thinking levels was carried out by matching students' answers and explanations to the indicators of each Van Hiele level. **Findings**: The study's results showed a significant decline in the achievement of each level of geometric thinking. All students reached the visualization stage; 81.8% reached the analysis stage, 22.7% reached informal deduction, and only 9.09% reached formal deduction, while no students reached the rigor stage. The sharp decline from the analysis to the deduction stage indicates students' limited ability to connect the properties of shapes and reason logistically. Interviews revealed that the local cultural context facilitated the visualization of shapes, but did not fully encourage higher deductive abilities. **Conclusion**: Students' geometric thinking skills are primarily at the stages of visualization and analysis. The use of ethnomathematical contexts has the potential to act as an early cognitive bridge, facilitating students' recognition of shapes and their properties. Compared to standard geometry problems, ethnomathematical problems better facilitate initial conceptual understanding through their connection to students' cultural experiences, but their contribution to formal deduction remains limited.

Keywords: geometric thinking skills, van hiele's theory, ethnomathematics.

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INTRODUCTION

Geometric thinking skills are essential basic skills for every individual to understand, analyze, and solve various problems (Waluya, 2022; Žakelj & Klanèar, 2022). Individuals who develop geometric thinking skills can recognize size, shape, and spatial relationships in the field of geometry (Gilligan-Lee et al., 2022). This understanding enables them to analyze geometric properties in everyday environments (Jablonski & Ludwig, 2023). Furthermore, they are also able to develop methods for calculating area, perimeter, and volume, as well as solving problems related to real life (Postier, 2021). Geometric thinking skills also develop reasoning skills that enhance understanding of the relationships between geometric objects, thus enabling accurate visualization of their position and shape (Whitacre, 2025). Furthermore, research by Xu et al. (2025) indicates that spatial thinking skills, encompassing visualization, orientation, and mental rotation, have a significant predictive value for achievement in geometry and measurement.

Several studies have analyzed geometric thinking skills using Pierre Van Hiele's theory (Naufal et al., 2021; Ruslau & Dadi, 2025). According to Van Hiele (1986) someone learning geometry will go through five stages: (1) stage 0, Introduction (Visualization), where students recognize and name shapes; (2) stage 1, Analysis, when students are able to group shapes based on properties; (3) stage 2, Informal Deduction, where students make observations and think deductively about shapes and properties; (4) stage 3, Deduction, where students think abstractly about geometric properties; and (5) stage 4, Rigor, where students reason formally (Uygun & Güner, 2021; Celik & Yilmaz, 2022). Research Arnal-Bailera & Manero (2024) examines the relevance of this model for mapping students' geometric cognitive development in a global learning context. Although numerous studies have examined this stage, those integrating Van Hiele's theory with local cultural contexts remain very limited.

Currently, mathematics education, particularly geometry, faces significant challenges (Brijlall & Abakah, 2022; Kpotosu et al., 2024). Many students struggle to grasp geometric concepts, which are often perceived as abstract and difficult to relate to real-life situations (Juman, 2022; Sarkar et al., 2020). The lack of relevance between the material and its practical application in real-life situations makes it difficult for students to see how these concepts are applied (Gük & Somyürek, 2025). This, in turn, hinders students' mathematical understanding and reduces their interest and motivation to study mathematics further (Li et al., 2021; Schukajlow et al., 2023). This suggests the need for a learning approach that connects cultural experiences with mathematical concepts to enhance learning understanding (Kabuye, 2024). Therefore, it is important to find approaches that bridge the gap between mathematical concepts and real-life situations (Anwar et al., 2023; Misqa et al., 2024).

As a solution to the irrelevance of classroom material and its practical application in real life, researchers chose an ethnomathematics approach as a relevant alternative to address this gap. This approach links mathematical concepts to local culture and everyday life practices (Suharta et al., 2021; Prahmana, 2022; Simbolon, 2024). Research by Acharya et al. (2021) demonstrates that the application of ethnomathematics can enhance students' motivation and understanding of geometry by incorporating cultural contexts, such as traditional house architecture and traditional cloth motifs. Thus, ethnomathematics not only enriches the learning experience but also fosters appreciation for cultural diversity (Polman et al., 2021).

The integration of ethnomathematics and Van Hiele's theory reflects the assumption that cultural context can serve as a cognitive bridge to facilitate transitions between stages of geometric thinking. Through cultural context, students can visually recognize geometric shapes, analyze their properties, and progress to higher levels of thinking, as outlined in Van Hiele's stages of development. A study by Kaya & Keºan (2023) showed that integrating cultural context into geometry learning improves students' deductive thinking skills and facilitates transitions between Van Hiele's stages of thinking. Therefore, this approach warrants further exploration in the context of Indonesian education, which is characterized by rich cultural diversity.

This study aims to analyze students' geometric thinking skills on plane geometry based on Van Hiele's theory through an ethnomathematics approach. It is hoped that students will gain a deeper understanding of geometric concepts within a local cultural context (Hwang et al., 2024; Rahayu & El Hakim, 2021). Furthermore, this study also aims to identify the difficulties students face at each stage of geometric understanding. Therefore, this study is expected to contribute to the development of geometry learning that is contextual, culturally relevant, and appropriate to students' geometric thinking stages. In addition, the findings of this study are expected to provide insights for teachers and curriculum developers in designing instructional strategies that connect students' cultural experiences with the hierarchical progression of Van Hiele levels, thereby enhancing conceptual understanding and learning engagement in geometry.

■ METHOD

Participants

This study involved 22 eighth-grader students (aged 12–13) from a public junior high school in Majalengka Regency, West Java, Indonesia. The school was purposively selected because it had a medium level of academic achievement and had implemented the *Merdeka* Curriculum in mathematics learning. Therefore, it was considered representative for describing students' geometric thinking skills at the early stages of secondary education. This approach aligns with common practice in qualitative studies of mathematics education, where the selection of contexts and participants is based on information-rich cases that can provide an in-depth understanding of the phenomena being studied (Creswell & Creswell, 2017; Leavy, 2022)

The sampling technique employed was purposive sampling (Patton, 2002), given that students had already studied basic geometry topics, such as triangles, rectangles, and circles. Therefore, students had adequate communication skills to participate in in-depth interviews. The relatively small sample size aligns with the characteristics of descriptive qualitative studies, where the primary objective of the research is not statistical generalization, but rather in-depth exploration and interpretation of participants' thinking patterns (Given, 2008; Merriam & Tisdell, 2025). This approach was also used in similar international studies exploring Van Hiele's levels of thinking in secondary education contexts (Diler & Öner, 2019). Research by Young & Casey (2018) shows that limited sample sizes can yield meaningful findings when accompanied by in-depth qualitative analysis.

Research Design and Procedures

This study employed a descriptive qualitative case study design aimed at gaining an in-depth understanding of students' geometric thinking abilities based on Van Hiele's theory. This design was chosen because it allows exploration of learning phenomena in a real-world context and provides a detailed description of students' thinking processes without experimental intervention (Tandililing et al., 2025). The descriptive case study approach is also commonly used in mathematics education research to examine the development of geometric thinking in secondary school students (Kania et al., 2021).

The research was conducted over a period of two months, from September to October 2024. All research activities were conducted in accordance with ethical principles of educational research, including obtaining permission from the school, obtaining voluntary consent from participants, and ensuring the confidentiality of student identities throughout the data collection and reporting process. The research procedures were systematically designed, as summarized in Table 1.

Table 1. Research stages and procedures

Research Stage	Activity	Objective
Instrument preparation and development	1. Develop a diagnostic test based on Van Hiele's theory and the local ethnomathematics context. 2. Developing unstructured interview guidelines; validation by two expert lecturers in mathematics education and one mathematics teacher.	Ensure the suitability of the content and validity of the instrument.
Implementation of diagnostic tests	Giving tests to 22 grade VII students.	Identifying the level of geometric thinking based on Van Hiele indicators (visualization–rigor).
Initial analysis and selection of interview subjects	1. Grouping test results into high, medium, and low ability categories. 2. Select one student from each category as the interview subject.	Identify subjects for in-depth exploration through unstructured interviews.
Unstructured interviews	Conduct in-depth interviews in the form of free conversations that focus on how students reason, explain, and construct geometric concepts.	Exploring thinking patterns and relationships with Van Hiele's levels.
Data analysis and triangulation	1. Analyze data through the stages of simplification, presentation, and drawing conclusions. 2. Triangulate sources between test results and interviews.	Ensuring the credibility and reliability of research results.

Instruments

This study employed two primary instruments: a diagnostic test and an unstructured interview, both developed based on Van Hiele's five stages of geometric thinking: Visualization, Analysis, Informal Deduction, Formal Deduction, and Rigor (Van Hiele, 1986; Usiskin, 1982). The local ethnomathematical context was integrated into the instrument development to link geometric concepts to students' cultural experiences, thus supporting the research objective of examining geometric thinking skills (Wibawa et al., 2024). Although the Rigor Level is generally associated with advanced formal thinking skills, one item at this level was included to explore students' initial potential for axiomatic reasoning in the context of simple proofs relevant to geometry learning in junior high school. This approach was maintained to maintain fidelity to Van Hiele's theoretical framework and provide a comprehensive picture of the range of students' thinking abilities.

Instrument Validation

The research instrument was validated through expert judgment by two mathematics education lecturers and one high school mathematics teacher. The validation process encompassed three main aspects: (1) content validity, to measure the suitability of the test items to the geometric thinking ability indicators at each Van Hiele level; (2) construct validity, to examine the integration between the indicators, measurement objectives, and the ethnomathemati context; and (3) readability and clarity of the test wording, to ensure it aligns with students' language characteristics. The validators provided input, particularly regarding the clarity of geometric terminology and the appropriateness of the local cultural context for several test items. Based on these suggestions, wording revisions were made to clarify the instructions, adjust the complexity of the test items to the formal deduction level, and ensure alignment between the cultural context and the mathematical concepts being measured. Overall, the expert assessments indicated that all test items were relevant to the geometric thinking ability indicators and suitable for in data collection without requiring further conceptual revision.

Diagnostic Tests

The diagnostic test consists of five open-ended essay questions structured around real-life contexts and local ethnomathematics elements, such as geometric shapes in mosque architecture. The questions are designed in stages to represent students' developmental stages, from recognizing shapes to proving basic concepts. The mapping of the questions to Van Hiele's levels of thinking is shown in Table 2.

Table 2. Mapping of diagnostic test items to van hiele's geometric thinking levels

Van Hiele Levels	Indicator	Objective
Visualization Stage (Level 0)	Recognize shapes visually	Identifying flat shapes in the picture of a mosque.
Analysis Stage (Level 1)	Explain the properties of shapes	Describe the characteristics and differences in shape.
Informal Deduction Stage (Level 2)	Explain the relationships between constructs	Reasoning about the result of dividing the diagonals of a square.
Formal Deduction Stage (3)	Using logic in measurements	Connecting the perimeter of a square with small area units.
Rigor Stage (Level 4)	Simple formal proof	Proving the formula for the area of a square

Student responses were assessed using a graded analytical rubric with a score range of 10–30 points, taking into account indicators of geometric thinking ability as outlined in Van Hiele's theory. Higher scores indicated deeper levels of understanding and reasoning. The assessment results were used to group students into high, medium, and low categories, which served as the basis for selecting interview subjects.

Unstructured Interviews

Interviews were conducted in an unstructured, fluid manner, adapting the conversation's direction to the results of the students' diagnostic tests. The goal was to explore the causes of students' difficulties and errors in solving geometry problems and to explore their thinking processes at each Van Hiele level. Conversations focused on general themes, including shape recognition, understanding the properties of shapes, relationships between shapes, logical reasoning, and simple conceptual justification. Interviews were conducted individually, lasting 10–15 minutes, and recorded with the participants' permission. The recordings were then transcribed for qualitative analysis. The interviews served to deepen and verify the diagnostic test findings through data triangulation, thus gaining a comprehensive understanding of the patterns and sources of students' geometric thinking difficulties.

Triangulation was conducted by thematically comparing diagnostic test and interview results to examine the consistency of students' reasoning patterns at each Van Hiele level. If discrepancies were found between test and interview data, researchers explored the context of the answers and the students' reasoning to identify the conceptual meaning behind the discrepancies. In situations where test results indicated numerical answers that did not align with the concept, but interview data revealed underlying logical reasoning, researchers interpreted the discrepancy as a representational variation, not a conceptual error. This procedure ensured that the determination of each student's geometric thinking level was based on a comprehensive and methodologically valid triangulation understanding.

Data Analysis

Data obtained from diagnostic tests and interviews were analyzed qualitatively and descriptively using a thematic analysis approach. This approach was chosen because it enables the systematic identification of students' thinking patterns and difficulties, based on themes emerging from the test and interview data, in accordance with the study's objectives. The analysis of the diagnostic test results and interview transcripts was conducted in three stages: data reduction, data presentation, and conclusion drawing (Miles et al., 2018). In the reduction stage, students' answers were grouped into low, medium, and high ability categories based on their diagnostic test scores. Their thinking levels were then determined based on the Van Hiele indicators. From each category, one representative student was selected as the subject of an in-depth interview and assigned the codes "low," "medium," and "high." The presentation stage involved organizing the data into themes that represent the characteristics of reasoning at each level of thinking, while the conclusion stage involved interpreting the relationship between difficulties, strategies, and students' geometric thinking levels. Data validity was ensured through source triangulation, peer review, and audit trail documentation, allowing the analysis results to be consistently and credibly accounted for. These analysis steps were carried out to answer the research objectives, namely to analyze students' geometric thinking abilities and difficulties at each level of thinking based on Van Hiele's theory.

Limitations

This research is contextual and exploratory, involving 22 students from a junior high school in Majalengka Regency. The findings are not intended to be generalized, but rather to provide an in-depth understanding of the variations in students' geometric thinking abilities. The instrument was developed based on Van Hiele's theory, with adaptations to the cognitive characteristics of junior high school students. Items at the Rigor level were included exploratively to assess the initial potential for axiomatic reasoning. Furthermore, unstructured interviews allowed students to express their natural responses, but the interpretation of the results remained dependent on their verbal abilities. Data validity was maintained through source triangulation, peer review, and an audit trail. Despite these limitations, this research still provides theoretical and practical contributions to the understanding of geometric thinking development. It serves as a foundation for further research with a more diverse scope and approach.

■ RESULT AND DISCUSSION

Five questions were given to 22 students who had studied plane geometry. The goal was to analyze students' achievement in geometric thinking skills based on Pierre Van Hiele's theory. Each question had a different level of difficulty according to Van Hiele's stage indicators. The overall results of student responses are presented as percentages, as shown in Table 3.

The descriptive results in Table 3 show a clear pattern of decline in students' geometric thinking skills across each Van Hiele level. All students (100%) reached the visualization level (Level 0), indicating a strong ability to recognize and identify the geometric shapes depicted in the mosque. Questions at this level required visual observation skills without requiring in-depth logical reasoning, allowing almost all students to answer correctly. At the analysis level (Level 1), the mean score decreased to 12.27 (SD = 5.23),

Table 3. Percentage of students' answers

Question Number	Stages of Geometric Thinking Skills according to Pierre Van Hiele's theory	Percentage	Maximum Score	Mean Score	Standard Deviation (SD)
1.	Visualization Stage (Level 0)	100%	10	10	0.00
2.	Analysis Stage (Level 1)	81.8%	15	12.27	5.23
3.	Informal Deduction Stage (Level 2)	22.7%	20	4.54	6.68
4.	Formal Deduction Stage (3)	9.09%	25	2.27	7.15
5.	Rigor Stage (Level 4)	0%	30	0	0.00

indicating that while most students were able to explain the basic properties of geometric shapes, they did not fully understand the relationships between these properties. Questions at this level required students to explain the characteristics and differences of shapes (e.g., parallel sides, side lengths, angles); however, many students simply memorized the characteristics without explaining the underlying mathematical rationale.

The overall trend in Table 3 demonstrates that students' performance declines progressively as the cognitive complexity of each Van Hiele level increases. This suggests that higher-order reasoning skills, such as connecting and justifying geometric relationships, are nt yet well-developed among most students. The sharp difference between Levels 1 and 2 reflects a cognitive transiti on gap, where learners move from recognizing and describing shapes to reasoning about their interrelationships. According to Van Hiele (1986), this transition necessitates carefully structured learning experiences that foster exploration, discussion, and conceptual reflection, rather than relying on rote memorization. The increasing standard deviation values at higher levels also suggest greater variation in students' reasoning ability, reinforcing the need for differentiated instructional approaches in geometry learning.

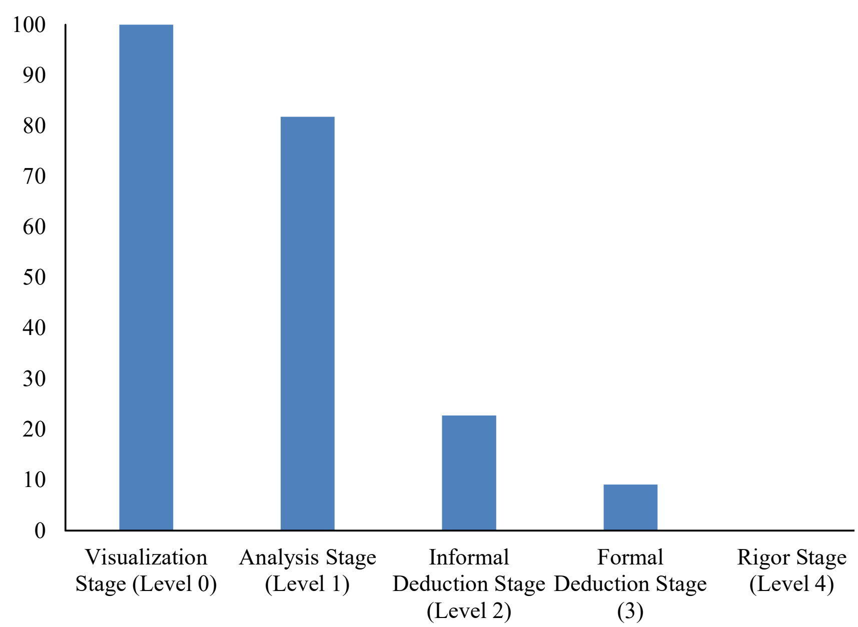


Figure 1. Percentage of students' answers based on van hiels' level

The sharpest decline occurred at the informal deduction level (Level 2) with a mean score of 4.54 (SD = 6.68). At this stage, students were asked to explain the shape formed by dividing the diagonals of a square and to reason about the relationships between shapes. Many students answered "isosceles triangle" without being able to explain the properties underlying the equality of its sides or angles. This indicates that the ability to connect properties between shapes and reason about logical relationships has not developed sufficiently. The data visualization in Figure 1 clarifies this finding, where the bar representing the analytical level (Level 1) appears significantly higher than the bar at the informal deduction level (Level 2). This sharp decline is an empirical indication that the transition from analytical thinking to relational-deductive thinking is the most critical stage in the development of students' geometric thinking.

This transition gap can be interpreted through Van Hiele's theoretical framework, which emphasizes that each level of geometric thinking requires a distinct language, set of relationships, and logical structure. Students operating at Level 1 tend to focus on recognizing and listing individual properties, whereas Level 2 demands the ability to coordinate and connect these properties into coherent logical reasoning. The difficulty observed here suggests that students' learning experiences are still oriented toward memorizing characteristics and classifying shapes, rather than engaging in conceptual reasoning and justification. Similar findings were reported by Burger and Shaughnessy (1986) and Fuys et al. (1988), who noted that students often remain at the visual or analytical levels when instruction emphasizes procedures over exploration and reflection. Within the present study, the ethnomathematical context of mosque architecture appears to have effectively supported visual recognition; however, it did not sufficiently stimulate students to build logical relationships or justify geometric properties, skills essential for entering the deductive stages.

A similar trend was seen at the formal deduction level (Level 3), where the mean score dropped to 2.27 (SD = 7.15). Questions at this level required deductive reasoning to relate the perimeter of a square to small units of area. Only a small proportion of students were able to consistently use measurement logic, while the majority still relied on procedural calculations without understanding the conceptual relationship between perimeter and area. The persistence of procedural reasoning suggests that formal deductive structures have not yet been fully internalized. According to Van Hiele's model, progression to Level 3 requires the ability to construct and manipulate logical arguments within a coherent system of relationships. However, many students' responses revealed dependence on memorized algorithms rather than conceptual understanding. The apparent stagnation between Levels 3 and 4 (as shown by the nearly flat pattern in Figure 1) indicates that students have not yet experienced instructional scaffolding that bridges conceptual understanding with formal proof reasoning.

At the rigor level (Level 4), no students answered the questions correctly (0%), with a mean and standard deviation of 0. Questions at this level required simple proofs of the formula for the area of a square, which demanded axiomatic thinking and formal reasoning skills. The absence of correct responses at this stage highlights the upper boundary of students' current geometric cognition. Advancement to the rigor level typically requires explicit guidance that encourages students to analyze the structure of geometric systems, recognize intertheoretical relationships, and justify statements through formal proofs. This finding reinforces the need to integrate reasoning- and proof-based instruction in middle-school geometry curricula, which often overemphasize procedural competence.

Overall, the bar graph in Figure 1 shows a systematic and progressive decline from level 0 to level 4, illustrating the increase in cognitive complexity as Van Hiele levels increase. The increasing standard deviation values at the intermediate and advanced levels also indicate greater variation in ability among students. This variation suggests that although all students can recognize geometric shapes (Level 0), only a few begin to develop relational and deductive reasoning. This phenomenon supports Van Hiele's (1986) assertion that progress in geometric thinking is discontinuous and depends on specific learning experiences rather than age. Consequently, geometry instruction should be intentionally structured to connect visualization, analysis, and deduction through guided inquiry, discussion, and conceptual reflection. In ethnomathematical contexts, cultural representations such as mosque architecture should not be limited to visual recognition. However, they should serve as conceptual bridges that help students construct logical reasoning and simple proofs.

Table 4. Work of high, medium, and low-level students

Subject	Stages of Geometric Thinking Skills according to Pierre Van Hiele's theory				
	Visualization Stage (Level 0)	**Analysis Stage (Level 1)**	**Informal Deductive Stage (Level 2)**	**Formal Deduction Stage (Level 3)**	**Rigor Stage (Level 4)**
High-level student work (27%)					
S1	✓	✓	✓	✓	X
S2	✓	✓	✓	X	X
S3	✓	✓	✓	X	X
S4	✓	✓	X	X	X
S5	✓	✓	✓	✓	X
S6	✓	✓	✓	X	X
Moderate level of student work (46%)					
S7	✓	✓	X		X
S8	✓	✓	X	X	X
S9	✓	✓	X	X	X
S10	✓	✓	X	X	X
S11	✓	✓	X	X	X
S12	✓	✓	X	X	X
S13	✓	✓	X	X	X
S14	✓	✓	X	X	X
S15	✓	✓	X	X	X
S16	✓	✓	X	X	X
Low-level student work (27%)					
S17	✓	X	X	X	X
S18	✓	X	X	X	X
S19	✓	✓	X	X	X
S20	✓	✓	X	X	X
S21	✓	X	X	X	X
S22	✓	X	X	X	X

The distribution shown in Table 4 reinforces the trend observed in Table 3, indicating that the higher the Van Hiele level, the fewer students are able to reach it. To visualize this relationship more clearly, Figure 2 presents a 100% stacked bar chart that displays the proportion of high-, medium-, and low-ability students who success fully achieved each stage of geometric thinking.

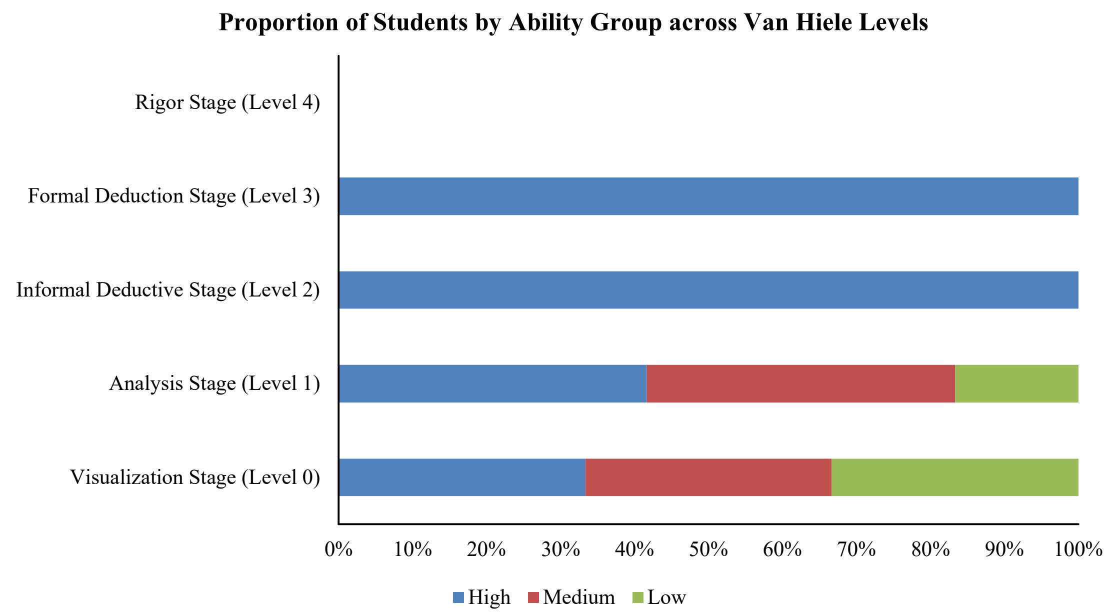


Figure 2. Proportion of high, medium, and low ability students achieving each van hiele level

Figure 2 illustrates a hierarchical yet uneven pattern of students' achievement across Van Hiele's levels of geometric thinking. All students (100%) successfully reached the visualization stage (Level 0), demonstrating strong recognition of shapes; however, the proportion dropped sharply at higher levels. Most students (73%) were concentrated in the visualization and analysis stages, while only a few high-ability students reached the informal and formal deduction stages. This steep decline between the analysis (Level 1) and informal deduction (Level 2) stages indicates a significant cognitive gap in transitioning from descriptive to relational reasoning.

The results suggest that while the ethnomathematical context of mosque architecture effectively facilitates visual identification and fundamental analysis, it does not sufficiently stimulate abstract or logical reasoning required for higher Van Hiele levels. Students tend to interpret geometric forms concretely rather than conceptualizing their underlying relationships. Hence, although the cultural context supports engagement and familiarity, it remains limited as a medium for fostering deductive reasoning and proof-oriented thinking.

To summarize the patterns observed in the diagnostic test, Table 5 presents a synthesis of students' geometric thinking characteristics and typical misconceptions across the Van Hiele levels, based solely on their written responses.

Table 5. Summary of students' geometric thinking characteristics and common misconceptions across van hiele levels (based on written test responses)

Van Hiele Level	Type of Misconception	Representative Example of Student Response
Level 0 (Visualization)	Identifying shapes based on appearance rather than properties.	Calling a rectangle "a long square" due to its visual shape.
Level 1 (Analysis)	Overgeneralizing attributes and failing to distinguish defining properties.	Claiming that all quadrilaterals with parallel sides are rectangles.

Level 2 (Informal Deduction)	Inability to relate geometric properties logically; reasoning remains descriptive.	Stating "a rhombus is not a square because it looks tilted."
Level 3 (Formal Deduction)	Applying formulas or procedures without understanding their logical basis.	Using the area formula correctly, but unable to explain or justify it.
Level 4 (Rigor Stage)	Unable to construct formal proofs or connect axiomatic relationships.	Knowing the formula for a square's area, but unable to prove it logically.

To gain a deeper understanding of the reasoning patterns and difficulties faced by students at each stage of geometric thinking, in-depth interviews were conducted with three students representing each ability category, namely S21 (low), S8 (medium), and S1 (high). The selection of these three students was based on the results of a diagnostic test that describes variations in the level of mastery of geometric concepts and thinking strategies. The interviews were conducted in an unstructured manner to explore how students reason, explaining the reasons behind their answers, and identifying difficulties that arise in understanding the concepts of plane geometry. A summary of the achievements of each subject at each stage of geometric thinking based on Van Hiele's theory is presented in Table 6 below.

Table 6. Achievement of students' geometric thinking skills based on the stages of pierre van hiele theory

Pierre Van Hiele Theory Skill Stages	S21 (Low)	S8 (Medium)	S1 (High)
Visualization Stage (Level 0)	✓	✓	✓
Analysis Stage (Level 1)	X	✓	✓
Informal Deduction Stage (Level 2)	X	X	✓
Formal Deduction Stage (3)	X	X	✓
Rigor Stage (Level 4)	X	X	X

Table 6 illustrates clear differences in achievement among the three subjects at each stage of geometric thinking, as outlined in Van Hiele's theory. All three were equally able to reach the visualization stage, but began to show variations at the analysis and deduction stages. To gain a deeper understanding of the reasoning characteristics and difficulties experienced by each subject, the following analysis is presented in detail based on each stage of thinking, from visualization to rigor.

Visualization Stage (Level 0)

In the visualization stage, students are asked to recognize various geometric shapes seen in architectural drawings of mosques. This cultural context is used to examine how students' visual experiences with familiar objects contribute to their intuitive identification of geometric shapes. The questions at this stage are presented in the form of architectural drawings of mosques, as shown in Figure 3.

Students' answers to this question showed variation in the number and types of geometric shapes they successfully identified. These differences reflect varying levels of accuracy and visualization abilities among the three subjects. A summary of the results for each subject is presented in Table 7 below.

1. Pay attention to the following image!

From the picture of the mosque above, there are various types of flat shapes. Can you name the shapes of the flat shapes in the picture?

Figure 3. Question number 1

Table 7. Students' answers to question number 1

Subject	Answer	Translate
S21	1. Persegi panjang, persegi, persegi segitiga	Rectangle, Square, Triangle
S8	1 segitiga, persegi panjang, Belah ketupat, persegi, layang-layang	Triangle, Rectangle, Rhombus, Square, Kite
S1	1. - Segitiga - Persegi - Persegi Panjang	Triangle Square Rectangle

To deepen the test results in the visualization stage, follow-up interviews were conducted with the three subjects. The goal was to explore how students recognize geometric shapes in mosque architecture and how this context helps them visually identify plane figures.

P : "Besides the geometric shapes you wrote in question number 1, do you see any other shapes in the picture of the mosque?"
S21 : "Nothing. I only see triangles on the roof and rectangles on the windows."
P : "So, was question number 1 easy or difficult?"
S21 : "Easy, because the shapes are immediately visible."
P : "How did you answer question number 1?"
S8 : "I can name five geometric shapes."
P : "What shapes are they?"
S8 : "Triangles, rectangles, squares, rhombuses, and kites. I saw them on the walls and decorations in the mosque."
P : "Was question number 1 easy or difficult?"
S8 : "Easy."
P : "Based on your answer to question 1, are there only three plane shapes?"
S1 : "Yes, because you only see three, like the triangle on the roof and the rectangle on the wall."
P : "Why do you call it a triangle?"
S1 : "Because two sides are the same length and meet at a point."
P : "Was question 1 easy or difficult?"

Test and interview results showed that the three students were able to recognize various flat shapes found in architectural drawings of mosques. At this stage, shape recognition was still based on overall visual observation without considering the properties or relationships between the elements of the shape, in accordance with the characteristics of the visualization stage (Level 0) in Van Hiele's theory.

S21 only mentioned basic shapes such as triangles, squares, and rectangles. This response suggests that shape recognition was performed directly based on external appearance, without considering geometric features. S8 was able to identify more shapes, including rhombuses and kites, found in the mosque's ornaments. This indicates that S8 has a broader visual attention to image details, although his recognition is still intuitive. S1 also recognized the same basic shapes, but he began to provide reasons by mentioning specific characteristics of the shapes he observed, such as two sides that are the same length and meet at a point. This indicates that S1 is not only relying on visual observation but is also starting to pay attention to the characteristics of the shapes, suggesting an early indication of the transition from the visualization stage to the analysis stage.

In general, the ethnomathematics context of mosque architecture plays a positive role in helping students recognize geometric shapes without requiring formal reasoning. Familiar shapes facilitate the observation process and strengthen their visualization skills. This aligns with Van Hiele's theory, as explained by Romano (2009), which posits that the development of geometric thinking occurs through meaningful and contextual learning experiences, helping students progress gradually from visualization to higher levels of analysis. Thus, all three students are at the visualization stage, but at different levels of ability: S21 is still at a basic level of recognition, S8 is beginning to notice variations in shapes, and S1 demonstrates higher visual accuracy and shows early indications of a transition to the analysis stage.

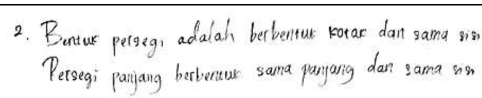
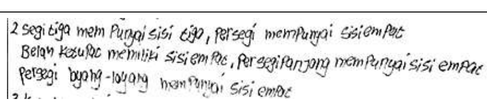
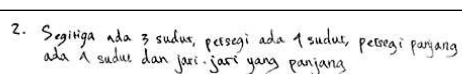
Analysis Stage (Level 1)

In the analysis stage, questions were designed to assess students' ability to recognize relationships between elements of plane shapes, such as side lengths, angles, and similar shapes, based on observations of mosque architecture. This cultural context was chosen so that students could connect previously visual shape recognition with simple reasoning about geometric characteristics.

Questions at this stage: *"Based on the plane shapes that you have obtained from question no. 1, explain the properties of plane shapes according to the shape of each plane shape."*

Students' answers show differences in how they explain the properties of recognized plane figures. A summary of the results from the answers of the three subjects is presented in Table 8.

Table 8. Students' answers to question number 2

Subject	Answer	Translate
S21		A square is a shape that resembles a box and has four equal sides. A rectangle has equal length and equal sides.
S8		A triangle has three sides, a square has four sides, a rhombus has four sides, a rectangle has four sides, a square has four sides.
S1		A triangle has three corners, a square has four corners, a rectangle has four corners and long radii.

To further deepen the understanding of the test results, interviews were conducted with the three subjects to explore how they comprehend and explain the properties of plane shapes recognized in the architectural drawings of the mosque.

P : *"How did you do problem number 2?"*
S21 : *"I only wrote down the properties of two plane figures."*
P : *"Why only two plane figures?"*
S21 : *"Because I was a bit unsure about the other figures, I forgot their properties a bit."*
P : *"Do you think problem number 2 was easy or difficult?"*
S21 : *"It was difficult, because I did not really remember the properties of the other plane figures."*
P : *"In question number 2, you wrote that triangles have three sides and squares have four sides. How did you know that?"*
S8 : *"I saw those shapes in a picture of a mosque. You can already see how many sides they have."*
P : *"Do you know any other properties of those shapes?"*
S8 : *"Not all of them, but I know that four sides are the same length, like a square."*
P : *"Did you find question number 2 easy?"*
S8 : *"Pretty easy, but I am not too sure about all of the properties."*
P : *"Of all the questions, which one was the most difficult?"*
S1 : *"Number 2."*
P : *"Why did you find number 2 difficult?"*
S1 : *"Because I have forgotten the properties of plane figures."*
P : *"But you wrote about the sum of the angles and diagonals, how did you know that?"*
S1 : *"I remember a little from class, that a rectangle has equal diagonals and four angles."*

Test and interview results indicate that students' abilities to explain the properties of plane figures vary. This difference illustrates the gradation of geometric thinking skills, from simply recognizing shapes understanding the properties of more logical shapes, as described in Van Hiele's theory at the analysis stage (Level 1).

S21 has not yet met the criteria for the analysis stage. He only wrote about squares and rectangles with general explanations such as "they are square" and "they have equal sides." These explanations suggest that S21 still understands shapes descriptively, rather than based on the relationships between geometric elements. In the interview, he admitted to forgetting the properties of other shapes and only writing down what he remembered visually. This indicates that S21 is still at the visualization stage (Level 0).

S8 began to demonstrate analytical skills by being able to name the number of sides of several shapes and recognize the similarity of side lengths in squares. These explanations indicate that S8 already understands some basic properties, although he has not yet explained the relationships between elements such as angle sizes or diagonals. Based on the interview, S8 connected his observations to architectural elements of the mosque, such as window shapes and wall ornaments, which helped him recognize simple properties of the shapes. Thus, S8 is at the initial analysis stage (Level 1).

S1 demonstrated a more complex understanding. He listed the number of angles and added diagonal elements to the rectangle, demons trating the ability to connect multiple properties into a logical structure. Although his language was not yet fully articulated, S1 demonstrated relational thinking between the elements of the shape, namely the relationship between sides, angles, and diagonals. Based on the interview, he recalled the concept of diagonals from the previous lesson, indicating that he has established analytical skills and is beginning to move towards the informal deduction stage (Level 2).

In general, the ethnomathematics context of mosque architecture helps students understand the properties of structures more concretely. Shapes such as roofs, windows, and ornaments provide visual references that make it easier for students to interpret the similarity of sides or the sum of angles. However, this context is not effective enough for low-ability students who remain fixated on visual observation without further reasoning, as explained by Ndlovu & Brijlall (2020), who note that students at the visualization stage only recognize shapes based on their appearance, not their properties. Thus, at this stage, S8 and S1 meet the characteristics of the analysis stage, while S21 has not yet demonstrated abilities appropriate to that level.

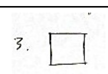
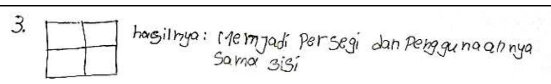
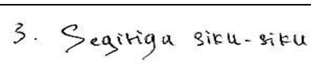
Informal Deduction Stage (Level 2)

The questions at this stage are presented in the form of pictures of mosque windows and wall ornaments containing various quadrilateral shapes, such as squares, rectangles, and rhombuses. Students are asked to explain the relationships between these shapes based on the properties of their sides and angles.

Questions at this stage: *"A square can be divided into two equal parts by drawing a diagonal line. What shape is formed by dividing the square along its diagonal? Explain the properties of the shape that is formed!"*

Table 9 presents a summary of the responses from the three subjects to informal deduction questions regarding the relationship between the properties of shapes formed by dividing a square.

Table 9. Students' answers to question number 3

Subject	Answer	Translate
S21		-
S8		The result becomes a square and its use is equal sides
S1		Right triangle

To deepen the test results, interviews were conducted with the three subjects to explore further how they understood the shapes formed and their relationship to the original structure.

P : *"Did you have any difficulty working on problem number 3?"*
S21 : *"Yes, I forgot about the diagonal line."*
P : *"So you did not have time to draw the diagonal?"*
S21 : *"Yes, I forgot that part."*
P : *"What do you think about question number 3?"*
S8 : *"It is a bit difficult, I do not really understand what it means."*
P : *"What part is difficult?"*
S8 : *"The question is about diagonal lines, and I do not understand the shape they form."*
P : *"What shapes are you studying?"*
S1 : *"Square… and right triangle."*
P : *"Do you think they are related?"*
S1 : *"Yes, if a square is divided diagonally, it becomes a right triangle."*

Test and interview results showed that only S1 displayed informal deductive thinking. He was able to reason that dividing a square by its diagonal produces two right-angled triangles and explain the relationship between them based on the properties of the diagonals. This understanding demonstrates the ability to logically relate the properties of shapes without formal proof.

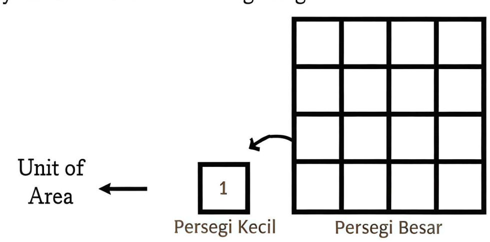
In contrast, S8 attempted to relate the division result to the property of equal sides, but did not yet understand the transformation that occurred. Although his answer was not yet correct, the attempt to explain the relationship between the shapes indicates that S8 is starting to move towards early deductive reasoning. Meanwhile, S21 only drew a square without adding a diagonal line or explaining the properties of the shape, indicating that he was still fixated on depicting visual shapes without geometric reasoning.

In general, the ethnomathematics context of mosque architectural ornaments helps students connect visual observations with geometric concepts. The forms found in mosque architecture facilitate students in recognizing similarities, differences, and relationships between shapes more concretely. However, only students with more mature analytical skills are able to reason deductively about the relationships between the properties of shapes. This aligns with Fuys et al. (1988) explanation that at the informal deduction stage (Level 2), students begin to understand the relationships between shapes and can put forward logical reasons for the interconnectedness of geometric properties. Thus, S1 has reached the informal deduction stage (Level 2), while S8 shows a tendency toward that stage, although it is still in the moderate category, and S21 remains at the analysis stage (Level 1).

Formal Deduction Stage (3)

Questions at this stage are designed to assess the extent to which students can draw deductive conclusions systematically and explain the reasons behind the relationships between shapes using the geometric concepts they have learned. Questions at this stage:

4. Please pay attention to the following image!



A large square has a side length of 4 units, calculate the perimeter of the square using small square units!

Figure 4. Question number 4

Table 10. Students' answers to question number 4

Subject	Answer	Translate
S21	-	-
S8	4. 8X8 = 64	The result of multiplying eight times eight
S1	4. 16 persegi kecil	There are sixteen small squares

Table 10 shows students' answers to question number 4, which requires them to determine the perimeter of a square using small square units as a reference.

To further analyze the test results, interviews were conducted with the three subjects to explore their understanding of the relationship between the perimeter of a square and the number of small square units it contains.

P : *"Why didn't you write down the answer to question number four?"*
S21 : *"I do not know what the question means, sir. I am confused about which part to count."*
P : *"Do you know what perimeter means?"*
S21 : *"Yes, perimeter is the sum of all the sides, but I do not know how it relates to the small squares inside."*
P : *"Did you try drawing or counting the sides?"*
S21 : *"No, because I am not sure where to start. I do not think this is a simple counting problem."*
P : *"So, do you think this question is easy or difficult?"*
S21 : *"It is difficult because I do not understand what it means, and I am afraid of answering it incorrectly."*
P : *"How did you do problem number four?"*
S8 : *"I saw a large square, so I counted eight sides, so it was eight times eight."*
P : *"Is the side of a square four?"*
S8 : *"Yes, but in the picture I see there are eight sides on each side."*
P : *"So that is how you calculated it?"*
S8 : *"Yes, I multiplied eight times eight, which was 64."*
P : *"How did you do problem number four?"*
S1 : *"I see the side of the square i four units, so there are 16 small squares in it."*
P : *"The question actually asks for perimeter, not area. Do you know the difference between the two?"*
S1 : *"Yes, I do. The perimeter is calculated by adding all the sides together, but I was thinking about the area first because it was easier to see the squares."*
P : *"Why did you use the side times side formula?"*
S1 : *"Because that way I know the number of small squares, which I can then relate to the length of the side. If one side is four squares, then the perimeter is four sides times four units."*
P : *"So you know what the perimeter is?"*
S1 : *"Sixteen units of length. So the area is 16 small squares, and the perimeter is also 16 units."*

S21 did not provide an answer due to difficulty understanding the question's intent and inability to connect circumference with area. He only recognized the concept of circumference descriptively without reasoning about the relationship between geometric elements. This indicates that S21 has not yet reached the formal deduction stage and is still at the analysis stage (Level 1). S8 wrote "8 × 8 = 64" without a logical explanation. He understood the calculation of area, but did not display the ability to explain the relationship between concepts deductively. His thinking pattern was still procedural and did not demonstrate logic-based reasoning. Therefore, S8 has not yet reached the formal deduction stage and remains at the final analysis stage (Levels 1 and 2).

S1 was able to explain the relationship between side length, the number of small squares, and the perimeter of a square. Although the problem only asked for the calculation of the circumference, he attempted to relate the concept of circumference to area by representing small square units. Her statement, "the area is 16 small squares and the perimeter is also 16 units," is not a conceptual equation, but rather a way for her to verify the regularity of the square shape and the consistency of measurements through visual representation. Therefore, S1 does not experience a misconception, but rather uses a visual strategy to reason about the logical relationship between side length, area, and perimeter. This indicates that S1 is not simply calculating but is reasoning about the relationships between geometric concepts using an early deductive approach. This way of thinking indicates that S1 has reached the early stage of Formal Deduction (Level 3), where students begin to understand the logical relationships between concepts and are able to generalize the properties of geometric shapes based on mathematical reasoning.

In the context of ethnomathematics, the use of square patterns in mosque architecture helps students visually recognize the regularity of shape and side proportions. However, at this stage, the cultural context is not yet sufficient to encourage axiomatic reasoning. S21 and S8 tend to get stuck on visual observation and simple calculations, while S1 begins to utilize the cultural context to understand the logical interconnections of concepts. Thus, only S1 reaches the formal deduction stage (Level 3). This finding aligns with Battista's (2007) assertion that the development of geometric thinking occurs gradually, progressing from visual recognition to formal deduction through structured conceptual experiences.

Rigor Stage (Level 4)

Questions at this stage require students to prove the formula for the area of a square logically based on the properties of squares.

Questions at this stage: *Given a square with a side length of 4 units, as in question number 4. Prove that the area formula is side × side."*

To illustrate students' responses at the rigorous stage, a summary of the answers from the three subjects is shown in Table 11.

Table 11. Students' answers to question number 5

Subject	Answer	Translate
S21	-	-
S8	-	-
S1	5. s × s = 4 × 4 = 16	5. s × s = 4 × 4 = 16

P : *"Why didn't you write down the answer to question number five?"*

S21 : *"I do not know how, sir. I have not memorized the formula for the area of a square yet."*

P : *"How do you prove that the area of a square is one side times one side?"*

S8 : *"I know the formula is one side times one side, but I cannot explain or prove why that is."*

P : *"How do you prove that the area of a square is one side times one side?"*

S1 : *"Because all sides are the same length. If the side is four units, that means there are four rows and four columns of small squares, so the area is 4 × 4. I know the formula, but I forgot how to prove it."*

S21 does not yet understand the concept of area and does not even know the formula, so he is still at the analysis stage (Level 1). S8 already knows the formula for the area of a square, but cannot explain the mathematical reasoning behind it. He only memorizes and applies the formula without logical reasoning, indicating that he is still at the informal deduction stage (Level 2). S1 wrote *"s × s = 4 × 4 = 16"*, which is an application of the formula, not a proof. Although he understands that all sides of a square are the same length and can connect the concept of rows and columns with area, the interview showed that he only knows the result of the formula without being able to construct a logical proof. This indicates that S1 has not reached the Rigor stage, but is at the upper limit of the Formal Deduction stage (Level 3), understanding the relationship between concepts and the importance of proof, but is not yet able to do it systematically.

In the context of ethnomathematics, the square pattern in mosque architecture helps students, especially those in S1, visually recognize the regularity of shape and side proportions; however, it is not enough to stimulate the axiomatic reasoning required at the rigorous stage. Students still interpret cultural forms as concrete representations, not as formal systems that can be proven mathematically. Thus, no students reached the rigor stage (Level 4). S1 showed a tendency to approach that stage, while S8 and S21 remained at a lower level of thinking, where the cultural context serves as a visual aid, not a formal basis for proof. This finding is consistent with Van Hiele (1986), which emphasizes that rigorous thinking skills can only be achieved through mastery of axiomatic structures and the development of mature formal proofs.

These results also provide important insights into the role of ethnomathematical context in students' geometric reasoning. Although the mosque's architectural context effectively supports students' visual and analytical understanding, it is insufficient to stimulate formal deductive reasoning. Cultural representations emphasize symmetry and aesthetic patterns that facilitate the recognition of geometric shapes but do not inherently represent logical relationships among their properties. As a result, students tend to focus on concrete visual features rather than engaging in abstract reasoning. These findings suggest that ethnomathematical contexts should be accompanied by reflective guidance and conceptual questions that help students connect cultural forms with mathematical logic, thus bridging the transition from descriptive observation to formal deductive reasoning.

CONCLUSION

The results of the study indicate that students' geometric thinking skills are still centered on the visualization and analysis stages. In contrast, informal and formal deduction skills are only achieved by a small number of students, and no students are able to reach the rigor stage. The ethnomathematical context of mosque architecture serves as a cognitive bridge in the early stages of learning, helping students recognize shapes and some of their properties through visual experiences that are close to their daily lives. However, this cultural context is insufficient to encourage higher deductive reasoning, such as explaining the relationships between elements of shapes, providing mathematical reasons, and conducting formal proofs. Thus, the main obstacle for students lies in the transition from the analysis stage to informal and formal deduction, which is characterized by a weak understanding of the relationships between geometric concepts.

This study is limited by its small sample size and the specific cultural context, so the findings cannot be broadly generalized. Based on the observed patterns of difficulty, it is recommended that ethnomathematics-based geometry learning be designed in stages to facilitate the development of students' reasoning skills, for example, through activities that require students to explain the reasons behind the properties of shapes (transition from Level 1 to Level 2) and connect concepts such as circumference and area in a consistent logical structure (transition from Level 2 to Level 3). Formal proof efforts can be introduced in the form of simple arguments based on local cultural patterns to help students transition to the rigorous stage. This approach is expected to strengthen the transition process of geometric thinking levels according to Van Hiele's theory.

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